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ABSTRACT

This report of the National Assessment of Educational Progress (NAEP) focuses on students' and teachers' reports about the classroom contexts for learning school mathematics in the top-performing one-third of schools compared to the bottom performing one-third. In 1990, 26,652 students from 1,237 schools participated in the study, while in 1992, 26,669 students from 1,582 schools were included. Data presented relate to program and curricular emphases, mathematics teachers and their classes, instructional approaches, calculators and computers, students' perceptions about mathematics, and students' mathematics course taking. Major findings were that in the two-year period from 1990 to 1992 the following signs of progress were noted: (1) Students reported taking more advanced coursework; (2) Teachers reported moving toward a more broadly-based curriculum; (3) Students and teachers reported more access to and use of calculators and computers; (4) Students were doing more daily problem solving from their textbooks; and (5) Students reported more positive attitudes about the value of mathematics. On the other hand, teachers continued to report discrepancies in resource availability and expectations between top- and bottom-performing schools, and teachers and students reported very little change in the frequency with which students were asked to engage in extended problem-solving activity. An appendix contains an overview of procedures used in NAEP's 1992 Mathematics Assessment. (MKR)

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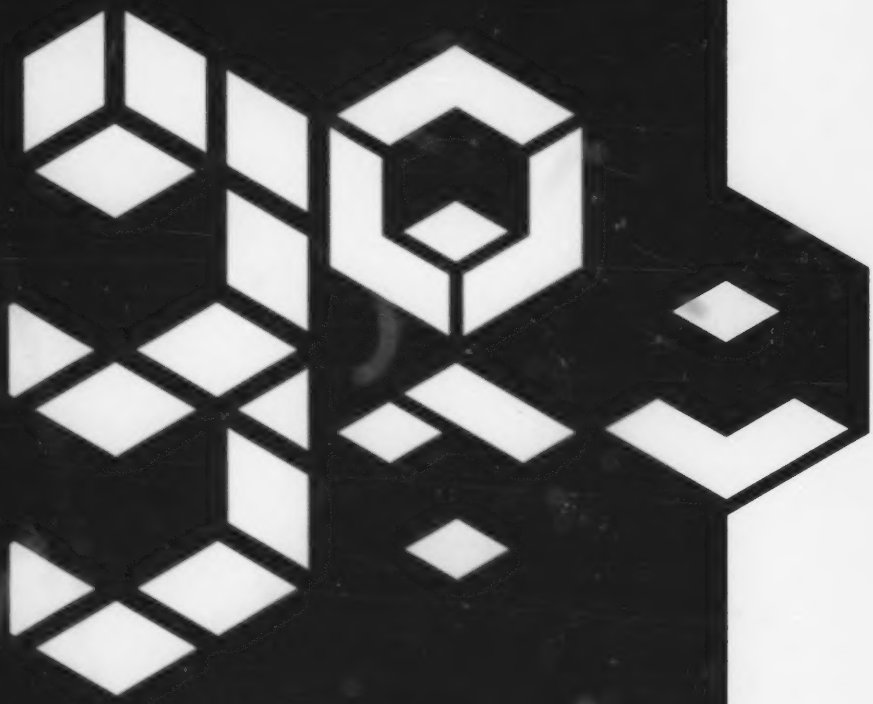
How School Mathematics Functions

Perspectives from the NAEP 1990 and 1992 Assessments



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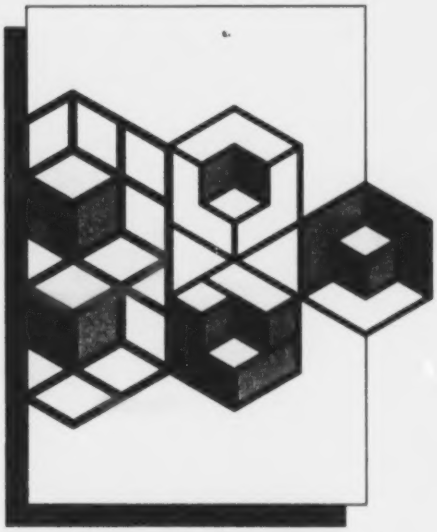

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Introduction

Mathematics education in the United States is now undergoing an impressive reformation. There have been numerous publications about how to improve it, most notably the *Curriculum and Evaluation Standards for School Mathematics* produced by the National Council of Teachers of Mathematics and various reports produced by the Mathematical Sciences Education Board, such as *Everybody Counts*.[1] These publications have addressed a number of important issues in mathematics reform, including improving the curriculum, adhering to high teaching standards, increasing the limited number of students — especially women and minorities — who pursue advanced coursework, providing more effective and meaningful instruction, and conducting more relevant assessments.

This report is the third in a series that discusses results from NAEP's 1992 mathematics assessment. In the *NAEP 1992 Mathematics Report Card for the Nation and the States,* NAEP reported that gains occurred between 1990 and 1992 across the nation at all three grades assessed — 4, 8, and 12.[2] There also were increases in average proficiency at grade 8 for 18 of the 37 states and territories that participated in the Trial State Assessment Program. At the same time, it also was noted that these gains represented only a modest step toward improving the state of mathematics achievement in our nation.

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[1] *Measuring What Counts: A Conceptual Guide for Mathematics Assessment* (Washington, DC: Mathematical Sciences Education Board and National Research Council, National Academy Press, 1993).

Most students, particularly at grades 8 and 12, showed success in basic addition, subtraction, and simple problem solving with whole numbers. Yet fourth graders had more difficulty solving two-step problems involving multiplication and division. Only about one-fifth and one-half of the students at grades 8 and 12, respectively, were estimated to have solved problems involving fractions, decimals, and percents as well as elementary concepts in geometry, statistics, and algebra.

The generally low levels of mathematics achievement observed for the nation and the states invited a more detailed look at students' problem-solving abilities. An analysis of student responses to questions requiring that they construct written answers showed that most students made a conscientious effort to respond, but the performances left much to be desired. In *Can Students Do Mathematical Problem Solving?*, NAEP published examples of extended problem-solving tasks and students' work on these tasks.[3] On problems requiring some depth of understanding and then explanation or illustrations of their solutions, the average percentage of students producing satisfactory or better responses was 16 percent at grade 4, 8 percent at grade 8, and 9 percent at grade 12. Results across the states paralleled these discouraging findings.

The focus in this report is on teachers' reports about the classroom contexts for learning school mathematics. In addition to measuring outcomes, students as well as their teachers and school administrators completed background questionnaires about a variety of home, community, school, and instructional factors. Students at grades 4, 8, and 12 answered questions about the instruction they received and their course taking. Also, teachers of the fourth and eighth graders who participated in the assessment provided information about school curricular practices, resource availability, and their instructional approaches. This report contains much of this information and provides the response percentages as well as the relationships between response categories and average proficiency. The data are presented for the top-performing one-third of schools (as judged by performance on the NAEP mathematics assessment) compared to the bottom-performing one-third of schools, and sometimes, for various demographic subpopulations. Trends between 1990 and 1992 are provided for much of the information.

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[2] Mullis, I.V.S., Dossey, J.A., Owen, E.H., & Phillips, G.W., *NAEP 1992 Mathematics Report Card for the Nation and States* (Washington, DC: National Center for Education Statistics, 1993).

[3] Dossey, J.A., Mullis, I.V.S., & Jones, C.O., *Can Students Do Mathematical Problem Solving?* (Washington, DC: National Center for Education Statistics, 1993).

Other reports also are available from NAEP's 1992 mathematics assessment. A Research and Development Report, *Effective Schools in Mathematics*,[4] focuses on the school questionnaire results collected from principals or their designees. It presents information about the characteristics of the top-performing one-third of schools compared to their bottom-performing counterparts as well as the results of hierarchical analyses examining the relationship between achievement and various school factors after adjusting for socioeconomic status. Although such adjustments are, for methodological reasons, always incomplete, this additional perspective helps underscore the role of some of the home and school factors examined by NAEP. A brief summary of findings from NAEP'S 1992 mathematics can be found in *America's Mathematics Problem: Raising Student Achievment*. Finally, a comprehensive set of tabular information covering the many aspects of the 1992 mathematics assessment can be found in the *Data Compendium for the NAEP 1992 Mathematics Assessment of the Nation and the States*.[5]

Major Findings

Although the two-year period between 1990 and 1992 is too short to establish trends, some signs of movement toward reform in school mathematics were noted. These changes in instructional context tend to support the gains in achievement documented in the *NAEP 1992 Mathematics Report Card for the Nation and the States*.[6]

- *Students reported taking more advanced coursework.* At grade 8, the percentages of students taking pre-algebra increased (from 20 to

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[4] Mullis, I.V.S., Jenkins, F., & Johnson, E.G., *Effective Schools in Mathematics* (Washington, DC: National Center for Education Statistics, 1994).

[5] *Data Compendium for the NAEP 1992 Mathematics Assessment of the Nation and the States* (Washington, DC: National Center for Education Statistics, U.S. Government Printing Office, 1993).

[6] Mullis, I.V.S., Dossey, J., Owen, E., & Phillips, G., *NAEP 1992 Mathematics Report Card for the Nation and the States* (Washington, DC: National Center for Education Statistics, 1993).

28 percent) while the percentages enrolled in a general eighth-grade mathematics class decreased (from 61 to 49 percent). Also, there was a decrease from 18 to 14 percent between 1990 and 1992 in the percentage of twelfth graders reporting three or fewer mathematics courses during high school.

- *Teachers reported moving toward a more broadly-based curriculum.* Although most students (90 percent at grade 4 and 76 percent at grade 8) were receiving heavy emphasis in numbers and operations, there were increases between 1990 and 1992 in the percentages of fourth graders receiving moderate emphasis in geometry and introductory algebra concepts, and more eighth graders were receiving at least a moderate emphasis in measurement and geometry.
- *Students and teachers reported more access to and use of calculators and computers.* In 1992, more fourth graders had access to school-owned calculators than in 1990 (59 compared to 44 percent). More eighth graders were permitted unrestricted classroom use of calculators, both generally and in testing situations. There were increases in computer access and use at grade 4.
- *Students were doing more daily problem solving from their textbooks.* Teachers at grades 4 and 8 as well as students at all three grades reported an increase in daily problem solving from textbooks. In 1992, teachers reported that 76 percent of the fourth graders and 83 percent of the eighth graders solved problems from textbooks on a daily basis. As a related finding, at grade 8, teachers reported assigning somewhat more mathematics homework in 1992 than in 1990, with fewer students (28 compared to 41 percent) given only 15 minutes of mathematics homework per night.
- *Students reported more positive attitudes about the value of mathematics.* More students in 1992 than in 1990 reported understanding the utility of mathematics to solve everyday problems and as part of job-related skills.

Still, not all the reports from teachers and students can be viewed positively.

- *Teachers reported discrepancies in resource availability and expectations between top- and bottom-performing schools.* Teachers reported that more students in bottom-performing than in top-performing one-third schools were in classrooms with only some or none of the necessary resources — 40 compared to 29 percent at grade 4, and 42 compared to 28 percent at grade 8. In 1992, 25 percent of the eighth graders in top one-third schools were expected to do 45 minutes of mathematics homework per night, compared to 16 percent in bottom one-third schools. The rather pervasive increase in calculator use at grade 8 was not observed in bottom-third schools.

- *Teachers and students reported very little change in the frequency with which students were asked to engage in extended problem-solving activity.* Teachers reported no increases between 1990 and 1992 in the percentages of students receiving heavy instructional emphasis in reasoning and communication for either grade 4 or grade 8. Only about half the students or fewer were receiving heavy instructional emphasis in these two areas emphasized in the *NCTM Standards*.[7] Teachers also reported that about half their students were never or hardly ever assessed using projects, portfolios, or presentations.

The percentage of students reporting that they were never or hardly ever asked to write reports or do mathematics projects increased significantly between 1990 and 1992 (from 70 to 77 percent at grade 8 and from 71 to 82 percent at grade 12). As a related finding, 62 percent of the grade 8 students and 68 percent at grade 12 reported that they never or hardly ever were asked to write a few sentences about how they solved a mathematics problem.

- *Despite a direct and powerful relationship between taking advanced mathematics courses and higher achievement, students reported an extremely low degree of mathematics coursework.* About half of the eighth graders (49 percent) were taking eighth-grade mathematics, while those with higher average proficiency were enrolled in pre-algebra (28 percent) or algebra (20 percent). Eighth graders planning to proceed to more advanced coursework (geometry or algebra I) in ninth grade also had higher average proficiency. However, 21 percent did not know what mathematics course they would take in grade 9.

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[7] National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 1989).

Only 42 percent of the twelfth graders reported taking eight semesters of mathematics coursework during their high school years. Twenty-three percent of the twelfth graders (36 percent in bottom-third schools) reported never studying geometry, which has become a "gatekeeper" course for access to higher education since most colleges are requiring this course prior to entrance. There were large discrepancies in the amounts of mathematics coursework reported among various subpopulations.

A Note on Interpretations

The selection of background questions included in the NAEP mathematics assessment was guided by the *NAEP 1992 Background Questionnaire Framework*.[8] In developing the framework, the committee of policy analysts and researchers considered the wide body of available research about factors including student learning as well as the particular purposes and strengths of NAEP data collection. In contrast to other questionnaires, which may ask for detailed information on a small set of topics, the 1992 NAEP questionnaires attempted to include a few well-targeted questions addressing a wide variety of topics. NAEP always has been sensitive to its voluntary nature and the burden it places on respondents. A deliberate effort has been made to curtail the questionnaires to about 15 to 30 minutes for any one respondent (students, of course, also spend approximately an hour on the achievement portion of the assessment). Thus, a limited pool of questions was available and the results necessarily must be considered in light of ongoing research about instructional practices.

When there are constraints, choices need to be made. In selecting the background questions associated with NAEP's 1992 mathematics assessment, three goals were kept in mind:

- to provide an educational context for understanding data on student achievement
- to identify differences in access to instruction and distribution of services for various types of students
- to track changes in policy-relevant variables across time

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[8] *National Assessment of Educational Progress, 1992 Background Questionnaire Framework* (Princeton, NJ: National Assessment of Educational Progress, Educational Testing, Service, 1992).

As much as possible, this report presents data in an attempt to address these goals. The survey data collected by NAEP can be used to monitor trends in key areas, while ongoing research can be used to determine the classroom contexts that best foster achievement. NAEP is able to ask questions that collect information about whether schools are implementing those strategies that work best to help students learn.

Because NAEP's is the only comprehensive set of data about school mathematics regularly collected at the national level, the NAEP information from teacher and student questionnaires has served the mathematics education community in a number of ways over the past 20 years. Topics of interest in this report include patterns in student course selection at the secondary level; the use and impact of hand calculators; and profiles of teachers' mathematical and pedagogical backgrounds. It also presents information on changes in the testing and assessment practices of teachers.

It is unwise, however, to rely too heavily on responses to any single question. Also, particularly given the reform environment, there undoubtedly is some inherently ambiguous language in the questions. Different terms have different meaning to different people, which can be further pronounced depending on differences in contexts and environments. Still, the data can be used to point practitioners toward examining their own program in greater detail. Perhaps more than anything, the data from the NAEP questionnaires can provide educators with a basis to confirm or illuminate patterns observed at the local level. They provide a back-up litmus test for checking the reasonableness of local findings. They also provide a source of questions and approaches for examining local policy issues, conducting local studies, and creating local initiatives to change practice.

Because of their basis in research, the NAEP survey results often help to confirm our understanding of how school and instructional factors relate to achievement. Although the effect of schooling and instruction are of prime concern, these analyses do not reveal the underlying relationships between background factors and performance. Associations between NAEP background factors and mathematics achievement must necessarily be interpreted cautiously given the correlational nature of NAEP data and the complexity of the context in which learning takes place. For example, associations may result because of differential instruction in which teachers tailor what they do based on their perceptions of students' abilities. That is, some instructional strategies may be more effective or appropriate for high-achieving students, while others are more suitable for lower-achieving students. In particular, various remedial techniques are often associated with lower average proficiency — not because *these strategies in any way* cause lower achievement, but because the poorer-performing students need special assistance.

Relationships between average proficiency and contextual variables are also affected by socioeconomic factors. These factors make it easier for wealthy school districts to hire better educated and more experienced teachers, and to provide more instructional materials than poorer districts, which must contend with poverty and its attendant social problems, both of which tend to depress student achievement.

In considering the findings reported herein, the reader should keep firmly in mind the high degree of interrelatedness among all the factors involved. It is impossible to ascribe cause and effect to single variables, in the sense that one single variable alone will result in higher achievement. The NAEP assessment results are most useful when they are considered in light of other knowledge about the education system, such as trends in instructional reform, changes in the school age population, and societal demands and expectations. Throughout this report, references are provided to assist the reader in finding additional related information about the topics covered.

1

Program and Curricular Emphases in School Mathematics

Recent studies comparing American mathematics education programs to those of other countries have found the United States to have less rigorous subject matter emphases and lower expectations for students.[9] Features of a solid mathematics curriculum include: a strong emphasis within the overall school curriculum, balance and continuity among the various mathematics content areas at differing grade levels, attention to the relative emphases placed on learning both concepts and procedures, and high expectations for student involvement in mathematics activities.[10] This chapter considers school, teacher, and student reports about aspects of the mathematics curriculum in American schools. NAEP data central to these considerations include information about the breadth of curriculum afforded students and the emphases given by teachers to the various content areas within the curriculum. Beyond the content and emphases placed on individual topics in the curriculum, the ways in which students are taught mathematics form the foundation for the inquiry, problem solving, and communication skills that students will need throughout their lives. The curriculum and its delivery can be viewed as the distribution system for the opportunity to learn mathematics. NAEP data pertinent to the teaching practices employed in mathematics classes are presented in subsequent chapters.

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[9] Lapointe, A. E., Mead, N. A., & Askew, J. M., *Learning Mathematics* (Princeton, NJ: Educational Testing Service, 1992).

Stevenson, H. W., Lummis, M., Lee, S., & Stigler, J. W., *Making the Grade in Mathematics: Elementary School Mathematics in the United States, Taiwan, and Japan* (Reston, VA: National Council of Teachers of Mathematics, 1990).

Stigler, J. W., Lee, S., & Stevenson, H. W., *Mathematical Knowledge: Mathematical Knowledge of Japanese, Chinese, and American Elementary School Children* (Reston, VA: National Council of Teachers of Mathematics, 1990).

Wirszup, I. & Streit, R., *Developments in School Mathematics Education Around the World (Vol. 3).* (Reston, VA: National Council of Teachers of Mathematics, 1992).

Mathematics As a School Priority

School principals (or their designees) were asked whether reading, writing, and mathematics received special priority in their curricula. The national data presented in Table 1.1 show that in 1992, schools reported mathematics as a special priority for 73 percent of grade 4 students and 67 percent of grade 8 students, which represents no significant change from 1990. However, mathematics did not receive significantly more emphasis than either reading or writing, unlike patterns observed in Japan and other Asian countries.[11] At grade 4, about the same priority was given mathematics in the bottom-performing one-third of the schools as in the top-performing one-third of the schools. (The top-performing and bottom-performing one-third of the schools were calculated by computing a mean for each school, then ranking and dividing the schools into thirds). This represented a significant decrease in priority for bottom one-third schools between 1990 and 1992. In 1992, at grade 8, mathematics, reading, and writing all received higher priority in bottom one-third schools than they did in top one-third schools.

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[10] Brooks, K. & Suydam, M., "Planning and Organizing Curriculum." In P. S. Wilson, *Research Ideas for the Classroom: High School Mathematics* (New York, NY: Macmillan, 1993).

[11] Stigler, J. W., & Perry, M., "Cross Cultural Studies of Mathematics Teaching and Learning: Recent Findings and New Directions." In D. A. Grouws, T. J. Cooney, & D. Jones, *Perspectives on Research on Effective Mathematics Teaching* (Hillsdale, NJ: Lawrence Erlbaum Publishing, 1988).

Table 1.1

Schools' Reports on the Identification of Mathematics as a Priority, Grades 4 and 8

	Assessment Years	Yes, Reading is a Special Priority	Yes, Writing is a Special Priority	Yes, Mathematics is a Special Priority
		Percentage of Students	Percentage of Students	Percentage of Students
Grade 4				
Nation	1992	83 (2.9)<	74 (3.2)	73 (3.1)
	1990	93 (2.6)	76 (3.4)	78 (4.1)
Top One-Third	1992	81 (5.5)	70 (4.9)	75 (5.7)
	1990	82 (7.1)	79 (7.8)	66 (9.2)
Bottom One-Third	1992	88 (4.0)	77 (4.8)	74 (5.4)<
	1990	100 (0.0)	73 (5.8)	92 (4.0)
Grade 8				
Nation	1992	64 (3.1)<	66 (3.4)	67 (3.3)
	1990	77 (5.1)	70 (5.0)	64 (5.4)
Top One-Third	1992	42 (6.0)	57 (5.3)	56 (6.9)
	1990	64(10.8)	62(10.6)	56(11.6)
Bottom One-Third	1992	80 (6.1)	75 (5.6)	75 (6.3)
	1990	78 (7.9)	67 (9.4)	74 (8.6)

>The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

<The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details).

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Instructional Time and Homework

Teachers were asked about the amount of time spent each week on mathematics instruction. To further bolster NAEP's information about the time devoted to mathematics, teachers and students also were asked to report the amount of student time required daily to complete homework assignments. Taken together, these data give a picture of the average time spent studying mathematics at grades 4 and 8 for American students.

Table 1.2 contains the teachers' reports on the amount of time spent weekly on mathematics instruction. The picture portrayed differed considerably between grades 4 and 8. At grade 4, teachers reported that 71 percent of the students received at least four hours of mathematics instruction per week. However, at grade 8, only 32 percent of the students received at least four hours of instruction per week. There was no clear relationship between amount of instructional time and average mathematics proficiency, partially because poorer students were receiving more instructional time than the better students, which may reflect efforts at remediation. A greater percentage of fourth graders in bottom one-third schools than in top one-third schools were receiving four or more hours of instruction per week.

Table 1.3 contains teachers' reports on the amount of daily homework assigned for students in grades 4 and 8. These data reflect a very consistent pattern of assigned homework for grade 4 students between 1990 and 1992. At grade 8, the data on teachers' reports reflect an increase in assigned homework, as fewer students, 28 percent in 1992 as compared to 41 percent in 1990, were being given only 15 minutes of homework per day. In the top third of the schools, 25 percent of the students were expected to do 45 minutes or more of homework per day compared to only 16 percent of the students in the lower one-third of the schools. The disparity between top and bottom one-third schools in the amount of homework assigned may be related to the type of mathematics courses taken. As Chapter Six explores in more depth, a majority of eighth graders in top one-third schools were taking pre-algebra or algebra courses, whereas most of those in bottom one-third schools were taking general mathematics courses. When the same question was analyzed by teachers' reports on the ability levels of students in their classes, the results for 45 minutes or more of homework were 40 percent for high-ability students, 17 percent for average-ability, and 9 percent for low-ability students.

Table 1.2

Teachers' Reports on the Amount of Time Spent on Mathematics Instruction Each Week, Grades 4 and 8

	TWO AND ONE-HALF HOURS OR LESS EACH WEEK		MORE THAN TWO AND ONE-HALF HOURS, BUT LESS THAN FOUR HOURS EACH WEEK		FOUR HOURS OR MORE EACH WEEK	
Assessment Year – 1992	**Percentage of Students**	**Average Proficiency**	**Percentage of Students**	**Average Proficiency**	**Percentage of Students**	**Average Proficiency**
Grade 4						
Nation	5 (0.8)	223 (3.3)	24 (1.8)	223 (2.0)	71 (2.1)	216 (1.1)
Top One-Third	6 (1.8)	232 (4.4)	33 (3.8)	238 (2.0)	60 (3.9)	235 (1.3)
Bottom One-Third	3 (1.0)	205 (6.1)	15 (2.8)	200 (4.4)	82 (2.9)	196 (1.6)
Grade 8						
Nation	13 (1.9)	269 (3.7)	55 (2.6)	270 (1.5)	32 (2.8)	267 (2.0)
Top One-Third	16 (4.4)	291 (4.5)	56 (5.4)	288 (1.9)	29 (6.1)	290 (2.5)
Bottom One-Third	11 (3.3)	245 (4.1)	52 (3.9)	248 (1.4)	37 (3.7)	245 (2.0)

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). The percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 1.3

Teachers' Reports on the Amount of Mathematics Homework Assigned Each Day, Grades 4 and 8

		NONE		15 MINUTES		30 MINUTES		45 MINUTES		AN HOUR OR MORE	
	Assessment Years	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Grade 4											
Nation	1992	6 (1.3)	221 (2.4)	52 (1.8)	220 (1.3)	37 (2.3)	217 (1.6)	4 (0.9)	201 (4.8)	1 (0.4)	206(11.6)
	1990	5 (0.9)	211 (3.4)	49 (3.1)	217 (1.7)	37 (3.3)	214 (2.0)	4 (1.1)	205 (6.0)	4 (1.2)	201 (8.6)
Top One-Third	1992	8 (2.6)	232 (2.4)	53 (4.3)	237 (1.5)	38 (5.8)	235 (2.0)	0 (0.5)	—	1 (0.5)	214 (5.5)
	1990	8 (3.4)	222 (4.9)	52 (4.8)	231 (2.7)	34 (5.1)	230 (2.6)	3 (1.6)	222 (1.9)	2 (2.4)	—
Bottom One-Third	1992	3 (1.2)	206 (5.1)	40 (3.3)	197 (2.5)	48 (3.5)	197 (1.6)	7 (2.5)	188 (4.9)	2 (1.1)	188 (7.8)
	1990	6 (3.1)	194(14.3)	43 (7.3)	198 (2.2)	40 (7.2)	194 (2.9)	4 (1.9)	182(11.1)	8 (2.3)	190 (3.6)
High Ability	1992	6 (3.9)	230(32.6)	60 (7.2)	241 (3.7)	32 (6.4)	233 (3.3)	3 (1.9)	233(18.5)	0 (0.0)	—
	1990	6 (2.5)	246 (3.9)	59 (9.9)	235 (6.3)	33 (9.3)	238 (7.8)	0 (0.0)	—	2 (2.1)	—
Average Ability	1992	7 (2.1)	223 (4.1)	50 (2.4)	224 (1.9)>	38 (3.5)	221 (1.8)	4 (1.5)	202 (4.8)	2 (1.0)	212(15.4)
	1990	4 (1.9)	214 (7.2)	45 (5.6)	216 (2.2)	46 (5.2)	216 (3.0)	4 (1.8)	190(11.7)	1 (0.8)	207(21.3)
Low Ability	1992	5 (2.5)	202(10.6)	59 (5.6)	198 (3.0)	28 (5.3)	191 (3.8)	7 (2.9)	189 (9.1)	1 (0.5)	207 (3.2)
	1990	7 (4.1)	192 (3.8)	37 (6.8)	203 (4.8)	34 (7.4)	200 (5.2)	10 (4.8)	211 (5.8)	11 (6.0)	211(25.5)
Mixed Ability	1992	5 (1.6)	224 (3.5)>	51 (3.4)	219 (1.3)	41 (3.5)	215 (2.1)	2 (0.8)	202 (8.0)	0 (0.3)	181(12.2)
	1990	6 (1.7)	206 (5.4)	54 (5.4)	215 (2.3)	33 (5.4)	211 (2.9)	3 (1.9)	216 (7.9)	4 (1.4)	195 (4.5)
Grade 8											
Nation	1992	3 (0.7)	238 (5.1)	28 (2.0)<	263 (1.7)	49 (2.5)	268 (1.4)	16 (1.9)	282 (3.4)	4 (0.8)	287 (5.0)
	1990	1 (0.3)	238 (8.3)	41 (3.8)	257 (2.3)	43 (3.9)	266 (2.5)	11 (1.8)	272 (4.8)	5 (0.9)	276 (5.7)
Top One-Third	1992	0 (0.2)	—	29 (3.8)	281 (1.4)	46 (4.3)	288 (2.4)	20 (4.4)	296 (4.0)	5 (1.7)	310 (3.3)
	1990	0 (0.3)	—	44 (6.2)	274 (2.3)	38 (5.9)	285 (2.8)	14 (3.4)	290 (4.5)	4 (2.2)	289(24.0)
Bottom One-Third	1992	6 (2.0)	236 (7.5)	25 (2.6)	239 (1.6)	53 (3.9)	249 (1.4)	12 (2.8)	258 (4.4)	4 (2.2)	256 (5.9)
	1990	1 (0.4)	217(20.7)	43 (7.2)	238 (3.1)	43 (6.5)	248 (4.1)	7 (1.9)	253 (8.4)	7 (1.7)	254(11.4)
High Ability	1992	0 (0.1)	—	11 (2.4)	300 (3.4)	49 (4.9)	297 (2.3)	30 (4.9)	302 (3.1)	9 (2.1)	304 (3.4)
	1990	0 (0.0)	—	16 (4.0)	288 (5.6)	51 (4.8)	290 (3.1)	18 (3.9)	292 (6.9)	14 (3.0)	280 (6.6)
Average Ability	1992	1 (0.5)	251 (8.1)	30 (3.1)	265 (2.3)	52 (3.7)	264 (1.5)	14 (3.2)	268 (4.2)	3 (1.4)	278 (9.3)
	1990	1 (0.4)	238(10.3)	44 (5.5)	259 (2.9)	45 (5.9)	259 (2.8)	10 (2.5)	270 (3.9)	1 (0.7)	286 (5.7)
Low Ability	1992	7 (2.3)	230 (4.6)	44 (4.5)	244 (3.4)	40 (5.0)	246 (2.1)	8 (3.4)	251(13.7)	1 (0.7)	220 (5.5)
	1990	3 (1.7)	239(12.7)	57 (6.0)	245 (4.2)	32 (5.4)	244 (5.6)	6 (3.0)	234(10.8)	2 (1.9)	233(29.9)
Mixed Ability	1992	6 (2.5)	239(10.4)	32 (4.5)	265 (2.2)	50 (5.3)	259 (2.8)	10 (3.7)	272 (7.2)	3 (1.0)	259 (9.9)
	1990	0 (0.0)	—	49 (9.3)	254 (5.3)	42 (9.3)	261 (3.4)	7 (3.4)	246(10.5)	2 (2.2)	—

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). When the proportion of students is either 0 percent of 100 percent, the standard error is inestimable. However, percentages 99.5 percent and greater were rounded to 100 percent and percentages 0.5 percent or less were rounded to 0 percent. Percentages may not total 100 percent due to rounding error.

— Sample size insufficient to produce a reliable estimate.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Students also were asked about the amount of mathematics homework they did on a daily basis. Table 1.4 reflects the responses given at grades 4, 8, and 12. At grade 4, lower percentages of students than teachers reported 15 or 30 minutes of daily homework, and higher percentages reported 45 or 60-plus minutes of homework. There was greater agreement at grade 8. But even here, teachers' estimates of the amount of daily assigned homework exceeded students' reports for the 30-minute interval, and 9 percent more students reported they had spent an hour or more on their homework.

At grade 4, the percentage of students reporting doing an hour or more of homework decreased from 15 to 12 percent between 1990 and 1992. At grade 8, more students, 36 versus 32 percent, reported doing 30 minutes of homework, an increase that appeared to come from the ranks who had previously done less than 30 minutes of homework daily.

At grade 12, reports about mathematics homework are provided for students currently enrolled in a mathematics class. Sixty-four percent of the twelfth graders reported that they were currently enrolled in a mathematics class, and they reported doing more homework than either the fourth or eighth graders. For example, in 1992, 18 percent of the twelfth graders taking mathematics reported 45 minutes of daily homework and 19 percent reported an hour or more.

The 1992 average proficiency data associated with the various amounts of homework indicated higher performance for grade 4 students reporting less than 45 minutes of homework than for those reporting 45 minutes or more. Because most sufficiently prepared fourth graders should be able to finish textbook exercise sets in elementary school textbooks in 45 minutes or less, students reporting more than 45 minutes of work per day may have been assigned extra drill/remediation work.

At grade 8, a different pattern emerged, as students reporting any homework had significantly higher mathematics proficiency levels than students reporting no homework. For twelfth graders currently taking mathematics, the pattern was the same as at grade 8. For both grades 8 and 12, there was little difference in the respective proficiency levels observed once students reported doing at least 15 minutes of homework per day.

Table 1.4

Students' Reports on the Amount of Time Spent on Mathematics Homework Each Day, Grades 4, 8, and 12

		NONE		15 MINUTES		30 MINUTES		45 MINUTES		AN HOUR OR MORE	
	Assessment Years	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Grade 4											
Nation	1992	6 (0.6)	222 (2.2)	40 (1.0)	221 (1.0)>	29 (0.7)	222 (1.0)>	12 (0.5)	217 (1.4)>	12 (0.6)<	205 (1.7)
	1990	7 (0.7)	220 (3.4)	39 (1.2)	217 (1.6)	27 (1.1)	217 (1.6)	12 (0.9)	206 (2.0)	15 (0.8)	201 (2.6)
Top One-Third	1992	6 (0.9)	243 (3.2)	44 (2.3)	237 (1.2)	30 (1.6)	237 (1.2)	12 (1.1)	236 (2.0)>	8 (1.0)	226 (2.9)
	1990	7 (1.4)	237 (5.6)	43 (2.1)	231 (2.2)	28 (1.9)	231 (2.5)	10 (1.5)	221 (4.4)	11 (1.6)	222 (4.0)
Bottom One-Third	1992	4 (0.5)	189 (4.3)	36 (1.6)	198 (1.6)	29 (0.8)>	202 (1.5)	12 (0.7)	194 (2.1)	17 (1.2)	190 (2.2)
	1990	7 (1.0)	197 (5.0)	37 (1.6)	196 (2.2)	24 (1.5)	200 (2.5)	13 (1.3)	192 (3.1)	18 (1.2)	186 (2.6)
Grade 8											
Nation	1992	8 (0.4)	255 (2.2)	28 (0.7)	270 (1.2)	36 (0.6)>	270 (1.3)>	16 (0.5)	270 (1.5)	13 (0.6)	266 (1.9)
	1990	9 (0.7)	252 (2.8)	31 (1.8)	265 (1.6)	32 (1.1)	264 (1.7)	16 (0.9)	266 (1.9)	12 (1.0)	259 (2.7)
Top One-Third	1992	4 (0.6)	280 (5.1)	29 (2.1)	289 (1.3)>	38 (1.3)	289 (2.1)	17 (1.3)	292 (2.4)	11 (1.4)	290 (3.3)
	1990	6 (1.2)	280 (4.6)	35 (4.7)	278 (1.5)	32 (2.4)	282 (2.6)	16 (1.5)	285 (3.3)	10 (2.0)	278 (4.3)
Bottom One-Third	1992	10 (0.9)	240 (3.6)	25 (0.9)	246 (1.7)	33 (1.0)	249 (1.2)	16 (0.7)	247 (1.8)	16 (1.0)	244 (2.4)
	1990	12 (1.6)	236 (3.8)	26 (1.8)	248 (3.0)	31 (1.7)	244 (2.4)	16 (1.3)	250 (3.2)	14 (1.5)	240 (3.8)
Grade 12†											
Nation	1992	9 (0.6)	295 (2.1)	22 (1.0)	306 (1.5)	32 (0.9)	306 (1.3)	18 (0.7)	310 (1.9)	19 (0.8)	310 (1.6)
	1990	10 (0.8)	293 (3.6)	21 (1.4)	304 (2.2)	30 (1.3)	305 (2.1)	17 (0.9)	307 (2.4)	21 (1.3)	306 (2.0)
Top One-Third	1992	7 (0.9)	316 (3.1)	22 (1.5)	318 (2.1)	32 (1.8)	321 (1.8)	20 (1.1)	323 (2.2)	19 (1.4)	329 (2.2)
	1990	9 (1.5)	314 (5.8)	22 (2.9)	312 (1.9)	33 (2.2)	319 (2.7)	17 (1.5)	326 (2.7)	19 (2.5)	322 (2.4)
Bottom One-Third	1992	12 (1.2)	276 (3.2)	19 (1.5)	289 (3.0)	30 (1.9)	284 (2.1)	16 (1.6)	285 (3.7)	22 (1.8)	284 (3.3)
	1990	13 (1.7)	273 (4.6)	16 (1.7)	279 (3.5)	25 (2.3)	281 (3.2)	18 (1.3)	278 (3.6)	28 (2.0)	285 (2.9)

† Sixty-four percent of the twelfth-grade students reported that they were taking a mathematics class at the time of the assessment.

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). The percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Instructional Emphases on Mathematics Content Areas

As a central component of NCTM's suggested reforms, *The NCTM Standards* propose shifting the mathematics curriculum away from a heavy focus on solely teaching arithmetic skills toward a program marked by balance among concepts and procedures, with an emphasis on the processes of reasoning, communicating, problem solving, and connecting mathematics with other areas of the human context.[12] To measure progress toward these goals and the underlying philosophies that may be guiding the curriculum, teachers were asked to estimate the emphasis placed on various mathematical content areas as well as on various types of mathematical ideas and skills.

Tables 1.5 through 1.9 provide teachers' reports at grades 4 and 8 about the degree of emphases placed on the five mathematics content areas delineated in the NAEP Mathematics Framework: Numbers and Operations; Measurement; Geometry; Data Analysis, Statistics, and Probability; and Algebra and Functions. Although there may be some ambiguity in interpreting the response scale "Heavy," "Moderate," and "Little or No" emphasis because one teacher's reading of "heavy" may be another's "moderate" and so on, some definite patterns emerge.

Numbers and Operations. The content area of numbers and operations encompasses students' knowledge of words, symbols, models, and conventions about numbers and operations, as well as their skill in employing that knowledge to perform standard algorithmic procedures using numbers.[13] Teachers reported placing a heavy emphasis on numbers and operations (see Table 1.5). Slightly more than 90 percent of the fourth graders and 76 percent of the eighth graders across the nation were receiving heavy instructional emphasis in this content area. At grade 8, teachers' reports suggest that students in high-ability classes (65 percent) were less likely to receive a heavy emphasis in numbers and operations than students in either average-ability (77 percent) or low-ability (86 percent) classes. Comparisons between 1990 and 1992 are not possible, because the questions about emphasis on numbers and operations were revised.

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[12] National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics* (Reston, VA: 1989).

National Council of Teachers of Mathematics, *Professional Standards for the Teaching of Mathematics* (Reston, VA: 1991).

[13] National Assessment of Educational Progress, *Mathematics Objectives: 1990 Assessment* (Princeton, NJ: Educational Testing Service, 1988).

Table 1.5

Teachers' Reports on the Instructional Emphasis Placed on Numbers and Operations, Grades 4 and 8

	HEAVY EMPHASIS		MODERATE EMPHASIS		LITTLE OR NO EMPHASIS	
Assessment Year – 1992	Percentage of Students	Numbers and Operations Proficiency	Percentage of Students	Numbers and Operations Proficiency	Percentage of Students	Numbers and Operations Proficiency
Grade 4						
Nation	92 (1.2)	218 (1.0)	8 (1.2)	219 (2.2)	0 (0.1)	—
Top One-Third	92 (2.2)	236 (1.2)	8 (2.2)	234 (3.0)	0 (0.0)	—
Bottom One-Third	91 (2.2)	196 (1.4)	9 (2.2)	200 (3.7)	0 (0.0)	—
High Ability	98 (1.5)	238 (2.6)	2 (1.5)	247 (2.5)	0 (0.0)	—
Average Ability	90 (2.2)	222 (1.3)	10 (2.2)	219 (3.2)	0 (0.2)	—
Low Ability	91 (3.4)	194 (2.0)	9 (3.4)	212 (6.1)	0 (0.0)	—
Mixed Ability	92 (1.8)	216 (1.3)	8 (1.8)	220 (3.7)	0 (0.0)	—
Grade 8						
Nation	76 (1.8)	267 (1.1)	21 (1.6)	273 (2.2)	3 (0.7)	281 (7.6)
Top One-Third	77 (3.0)	287 (1.9)	19 (3.0)	292 (2.3)	4 (1.6)	306 (6.1)
Bottom One-Third	79 (3.4)	246 (1.1)	18 (2.6)	252 (3.0)	3 (1.4)	235 (3.1)
High Ability	65 (3.4)	299 (2.2)	28 (3.3)	299 (3.1)	7 (1.7)	308 (5.4)
Average Ability	77 (3.3)	266 (1.5)	21 (3.1)	267 (2.6)	2 (1.1)	249 (7.0)
Low Ability	86 (2.4)	245 (2.1)	13 (2.3)	237 (4.6)	1 (0.4)	252 (7.6)
Mixed Ability	78 (3.3)	260 (1.7)	20 (3.5)	267 (2.3)	2 (1.2)	243(11.3)

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). When the proportion of students is either 0 percent or 100 percent, the standard error is inestimable. However, percentages 99.5 percent and greater were rounded to 100 percent and percentages 0.5 percent or less were rounded to 0 percent. In 1990, different procedures which involved subtopics were used to determine the emphasis placed on numbers and operations .

— Sample size insufficient to permit a reliable estimate.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Measurement. The national data for grades 4 and 8 (in Table 1.6) reflect considerably less emphasis being given to topics in measurement than to topics in numbers and operations. Teachers reported that 13 percent of the grade 4 students and 16 percent of the grade 8 students were receiving a heavy emphasis in measurement. An additional 81 percent of the fourth graders were receiving a moderate emphasis in measurement, an increase of 11 percent from 1990.

At grade 8, a substantial upward shift between 1990 and 1992 occurred in the percentage of students receiving moderate emphasis in measurement. In 1992, 69 percent of the students were receiving a moderate emphasis in measurement, compared to 50 percent in 1990. This increase was concomitant with a significant decrease in the percentage of students, from 33 to 16 percent, receiving little or no emphasis in measurement. There were significant increases in the percentages of students in high-, average-, and low-ability classes receiving a moderate emphasis in measurement — from 32 to 58 percent, from 58 to 72 percent, and from 52 to 74 percent, respectively. Each of these three increases was accompanied by a significant decrease in the percentage of students receiving little or no emphasis in measurement. This pattern of change was also noted in the percentages of students in the top- and bottom-performing one-third of the schools. The percentage receiving moderate emphasis increased, from 42 to 70, for those students in the top one-third of the schools, and from 49 to 72 percent for students in the lowest one-third of the schools. In both cases, these increases were paralleled by similar decreases in the percentages of students receiving little or no emphasis in measurement.

Table 1.6

Teachers' Reports on the Instructional Emphasis Placed on Measurement, Grades 4 and 8

		HEAVY EMPHASIS		MODERATE EMPHASIS		LITTLE OR NO EMPHASIS	
	Assessment Years	Percentage of Students	Measurement Proficiency	Percentage of Students	Measurement Proficiency	Percentage of Students	Measurement Proficiency
Grade 4							
Nation	1992	13 (1.5)	215 (2.2)	81 (1.6)>	219 (1.2)>	6 (1.1)	217 (3.1)
	1990	19 (3.1)	214 (3.3)	70 (3.9)	214 (1.1)	12 (2.8)	217 (2.9)
Top One-Third	1992	10 (2.5)	233 (2.9)	85 (3.2)	236 (1.2)>	5 (2.1)	235 (4.1)
	1990	14 (6.2)	235 (7.4)	70 (7.1)	230 (1.7)	16 (4.9)	225 (3.5)
Bottom One-Third	1992	21 (3.6)	199 (2.1)	76 (3.9)	196 (1.6)	2 (1.1)	179 (3.5)
	1990	28 (5.8)	197 (2.8)	65 (5.9)	194 (2.2)	7 (3.4)	194 (6.0)
High Ability	1992	13 (5.3)	244 (6.1)	78 (6.9)	237 (3.3)	9 (4.9)	238 (8.7)
	1990	17 (6.2)	238(11.5)	71 (9.7)	238 (4.9)	12 (8.1)	224(13.4)
Average Ability	1992	13 (1.8)	216 (3.1)	85 (1.9)	222 (1.4)>	2 (0.8)<	219 (4.2)
	1990	16 (4.4)	208 (5.6)	74 (5.1)	215 (2.2)	10 (3.1)	226 (4.6)
Low Ability	1992	9 (2.7)	192 (6.9)	81 (3.6)	196 (2.3)	10 (2.8)	200 (6.0)
	1990	12 (4.5)	195(10.3)	76 (7.3)	204 (4.2)	12 (6.4)	200(11.6)
Mixed Ability	1992	16 (3.1)	212 (2.5)	76 (3.7)	218 (1.6)>	8 (2.7)	218 (3.6)
	1990	19 (6.0)	213 (6.7)	66 (6.1)	212 (1.7)	15 (4.3)	214 (4.5)
Grade 8							
Nation	1992	16 (1.8)	260 (2.0)	69 (1.9)>	267 (1.3)>	16 (1.5)<	283 (2.4)
	1990	17 (2.8)	255 (3.5)	50 (3.6)	260 (1.9)	33 (3.7)	274 (3.1)
Top One-Third	1992	6 (1.8)	284 (4.7)	70 (3.7)>	287 (2.3)>	24 (3.4)<	296 (2.9)
	1990	15 (4.0)	274 (4.9)	42 (5.4)	274 (2.7)	44 (5.7)	290 (2.1)
Bottom One-Third	1992	20 (3.8)	245 (2.2)	72 (3.9)>	247 (1.2)	8 (1.8)<	252 (4.6)
	1990	24 (6.7)	238 (3.4)	49 (6.9)	242 (3.0)	27 (6.4)	254 (4.4)
High Ability	1992	10 (2.2)	290 (4.7)	58 (3.2)>	300 (2.3)>	33 (3.2)<	302 (2.8)
	1990	14 (4.1)	270 (7.2)	32 (5.0)	290 (3.1)	54 (6.2)	295 (3.0)
Average Ability	1992	18 (2.5)	260 (3.7)	72 (2.7)>	266 (1.4)>	10 (1.5)<	274 (3.4)
	1990	15 (3.6)	259 (5.7)	58 (5.0)	257 (2.6)	27 (4.3)	268 (4.6)
Low Ability	1992	16 (3.8)	245 (7.3)	74 (5.4)>	244 (2.3)	10 (3.0)<	247 (7.1)
	1990	20 (4.8)	239 (4.4)	52 (6.5)	243 (4.3)	28 (6.3)	249 (6.2)
Mixed Ability	1992	17 (3.8)	256 (3.3)	72 (3.8)	262 (2.3)	11 (2.7)	266 (3.7)
	1990	22 (8.0)	251 (7.7)	54 (9.0)	258 (4.5)	25 (7.2)	259 (4.2)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Geometry. The national data in Table 1.7 indicate that the same percentages of grade 4 and grade 8 students — 71 percent — were receiving moderate instructional emphasis in geometry. However, 18 percent of the grade 8 students were receiving heavy emphasis, compared to only 6 percent of the grade 4 students. At both grades, the percentage of students receiving at least moderate instructional emphasis in geometry significantly increased from 1990 to 1992. The increases mirrored similar significant drops in the percentages of students receiving little or no emphasis in geometry.

For students in average-ability classrooms at both grades 4 and 8, there were increases in the percentages receiving a moderate emphasis in geometry and related decreases in the percentages receiving little or no emphasis. At grade 8, there also were increases in the percentages of students in high-ability and low-ability classrooms who received a moderate emphasis in geometry. The dramatic increase that occurred in low-ability classrooms may reflect a change that has been occurring in the geometry curriculum. In recent years, less formal geometry courses that emphasize application in real-world settings have been introduced alongside college preparatory geometry classes.

Table 1.7

Teachers' Reports on the Instructional Emphasis Placed on Geometry, Grades 4 and 8

		HEAVY EMPHASIS		MODERATE EMPHASIS		LITTLE OR NO EMPHASIS	
	Assessment Years	Percentage of Students	Geometry Proficiency	Percentage of Students	Geometry Proficiency	Percentage of Students	Geometry Proficiency
Grade 4							
Nation	1992	6 (0.9)	211 (4.1)	71 (2.6)>	219 (1.1)>	23 (2.5)<	216 (1.7)
	1990	8 (1.8)	205 (4.1)	58 (3.3)	215 (1.1)	34 (3.3)	215 (2.1)
Top One-Third	1992	4 (1.3)	233 (4.5)	77 (4.8)	237 (1.2)>	19 (4.5)<	232 (3.3)
	1990	2 (1.4)	240(15.4)	58 (6.8)	230 (1.7)	40 (6.9)	228 (3.3)
Bottom One-Third	1992	10 (2.5)	196 (2.7)	68 (4.4)	196 (1.8)	21 (4.2)	198 (2.5)
	1990	18 (5.4)	196 (2.4)	53 (5.0)	195 (2.3)	29 (4.8)	196 (2.9)
High Ability	1992	8 (4.4)	238 (8.0)	71 (7.3)	237 (2.2)	21 (6.0)	239(12.0)
	1990	12 (4.7)	236(16.8)	60(10.1)	235 (5.3)	28(11.1)	239(10.8)
Average Ability	1992	6 (1.5)	210 (4.4)	73 (3.8)>	224 (1.5)>	20 (3.6)<	218 (2.3)
	1990	7 (3.1)	196 (5.4)	58 (5.1)	214 (2.2)	35 (5.0)	220 (2.8)
Low Ability	1992	4 (1.6)	186 (7.2)	67 (4.7)	195 (3.3)	29 (4.7)	200 (3.8)
	1990	5 (3.5)	202(18.5)	60 (6.4)	203 (5.3)	35 (8.1)	200 (5.4)
Mixed Ability	1992	6 (1.8)	210 (6.6)	69 (3.5)	218 (1.5)	25 (3.9)	216 (2.1)
	1990	5 (1.9)	198 (7.0)	56 (6.3)	215 (2.4)	39 (6.2)	211 (2.4)
Grade 8							
Nation	1992	18 (2.4)	269 (2.3)	71 (2.6)>	268 (1.3)>	11 (1.4)<	272 (4.2)
	1990	28 (3.6)	262 (2.7)	51 (4.0)	263 (1.7)	22 (3.1)	268 (4.9)
Top One-Third	1992	15 (3.0)	291 (2.9)>	72 (3.5)>	287 (1.9)>	13 (2.3)	296 (4.1)
	1990	29 (6.2)	276 (3.4)	54 (5.7)	278 (1.5)	18 (3.0)	297 (4.3)
Bottom One-Third	1992	20 (3.2)	249 (2.3)	71 (3.3)	247 (1.2)	9 (1.9)	237 (6.8)
	1990	27 (6.4)	247 (5.7)	50 (8.5)	244 (2.7)	23 (6.6)	241 (4.7)
High Ability	1992	17 (2.8)	293 (3.3)	60 (3.8)>	301 (2.4)>	23 (2.6)	300 (3.0)
	1990	23 (5.0)	280 (5.0)	42 (5.6)	285 (3.6)	35 (5.6)	299 (4.1)
Average Ability	1992	20 (3.7)	268 (3.4)	74 (3.9)>	265 (1.6)	6 (1.4)<	265 (4.6)
	1990	24 (4.9)	261 (6.1)	56 (5.0)	262 (2.2)	19 (4.6)	254 (5.8)
Low Ability	1992	15 (3.8)	246 (5.9)	77 (4.0)>	246 (2.5)	8 (1.9)	219 (3.9)<
	1990	30 (6.5)	240 (4.0)	46 (6.6)	250 (5.1)	24 (6.4)	238 (5.9)
Mixed Ability	1992	16 (3.8)	259 (2.5)	74 (4.2)	264 (1.8)	10 (3.0)	244 (8.8)
	1990	36 (8.4)	263 (4.2)	54 (8.1)	255 (4.6)	10 (3.3)	243 (7.8)

>The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

<The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Data Analysis, Statistics, and Probability. *The NCTM Standards* recommend that the area of data analysis receive special attention in the K-8 curriculum. K-8 textbooks have added chapters on statistics and probability,[14] and teachers have participated in summer or in-service coursework in the area. Although only 7 percent of the grade 4 students and 11 percent of the grade 8 students received what their teachers characterized as a heavy emphasis in data analysis, 40 percent of the fourth graders and 60 percent of the eighth graders received moderate emphasis (see Table 1.8). Still, at grade 4, the question simply pertained to introductory concepts, and teachers reported little or no emphasis on this content area for the majority of students (54 percent). Thirty percent of the eighth graders were receiving little or no emphases in data analysis, probability, and statistics. The instructional emphasis placed on this content area did not vary much by top- and bottom-performing schools or by class ability level. Comparisons across the two-year period of time are not possible, because the questions about emphasis on data analysis, statistics, and probability were revised between 1990 and 1992.

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[14] Bright, G. W. & Hoeffner, K., "Measurement, Probability, Statistics, and Graphing." In D. T. Owens, *Research Ideas for the Classroom: Middle Grades Mathematics* (New York, NY: Macmillan, 1993).

Shaughnessy, J. M. & Bergman, B., "Thinking about Uncertainty: Probability and Statistics." In P. S. Wilson, *Research Ideas for the Classroom: High School Mathematics* (New York, NY: Macmillan, 1993).

Table 1.8

Teachers' Reports on the Instructional Emphasis Placed on Data Analysis, Statistics, and Probability, Grades 4 and 8

Assessment Year – 1992	HEAVY EMPHASIS		MODERATE EMPHASIS		LITTLE OR NO EMPHASIS	
	Percentage of Students	Data Analysis Proficiency	Percentage of Students	Data Analysis Proficiency	Percentage of Students	Data Analysis Proficiency
Grade 4						
Nation	7 (1.0)	222 (3.6)	40 (2.5)	220 (1.7)	54 (2.6)	216 (1.1)
Top One-Third	9 (2.5)	237 (4.1)	43 (5.0)	238 (2.0)	48 (5.1)	234 (1.4)
Bottom One-Third	7 (2.4)	200 (5.4)	40 (4.2)	194 (1.7)	52 (3.9)	198 (2.0)
High Ability	3 (1.9)	226(29.8)	51 (7.0)	243 (4.3)	46 (7.2)	233 (3.5)
Average Ability	7 (1.9)	227 (4.8)	41 (3.8)	224 (1.9)	52 (4.1)	219 (1.6)
Low Ability	2 (1.5)	205(12.5)	36 (3.9)	194 (3.9)	61 (4.1)	197 (2.5)
Mixed Ability	8 (2.0)	218 (4.0)	37 (3.7)	215 (2.4)	55 (3.9)	218 (1.9)
Grade 8						
Nation	11 (1.6)	273 (3.7)	60 (1.8)	268 (1.3)	30 (1.8)	267 (2.2)
Top One-Third	9 (3.2)	304 (4.7)	59 (3.0)	287 (2.1)	32 (3.3)	289 (2.8)
Bottom One-Third	10 (1.8)	248 (3.6)	55 (3.7)	247 (1.8)	34 (3.5)	246 (2.3)
High Ability	16 (3.6)	297 (5.7)	52 (3.3)	299 (2.1)	32 (4.4)	302 (2.2)
Average Ability	8 (1.5)	266 (4.5)	68 (2.9)	265 (1.5)	24 (2.9)	265 (2.9)
Low Ability	7 (3.1)	249 (7.3)	54 (5.5)	245 (2.5)	40 (5.6)	242 (3.6)
Mixed Ability	14 (4.0)	261 (3.2)	57 (4.5)	264 (2.3)	29 (4.9)	256 (2.9)

NOTE: The question specified informal introduction of concepts at grade 4.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error. In 1990, different procedures which involved subtopics were used to determine the emphasis placed on data analysis, statistics, and probability.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Algebra and Functions. Students' opportunity to learn algebraic material in grades K-8 has become a topic of national discussion, as mathematics educators and curriculum specialists compare the United States' mathematics curriculum to those of its international economic competitors.[15] The data in Table 1.9 indicate that algebra receives much less instructional emphasis at grade 4 than at grade 8.

Teachers reported that fourth graders received little exposure to introductory concepts in algebra and functions, with 66 percent receiving little or no emphasis in this content area. However, between 1990 and 1992, there was an increase in the percentage of fourth graders receiving moderate emphasis in introductory algebra concepts, from 17 to 30 percent. At the same time, the percentage of grade 4 students receiving little or no emphasis in algebra decreased from 82 to 66 percent.

Compared to 1990, a greater percentage of eighth graders in 1992 also may have received at least moderate exposure to algebra and functions content but the increase was not statistically significant. In both 1990 and 1992, most of the eighth graders in high-ability classes (87 to 92 percent) were receiving heavy emphasis in algebra, while relatively few of those in low-ability classes (18 to 19 percent) were receiving this emphasis. Because the shift toward emphasizing algebra in the middle grades is rather abrupt and the timing of the shift is differentiated according to mathematics ability, discontinuities can occur for middle school students learning algebraic material.[16]

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[15] McKnight, C. C., et al, *The Underachieving Curriculum: Assessing U. S. School Mathematics from an International Perspective* (Champaign, IL: International Association for the Evaluation of Educational Achievement, 1987).

Lapointe, A. E., Mead, N. A., & Askew, J. M., *Learning Mathematics* (Princeton, NJ: Educational Testing Service, 1992).

[16] Flanders, J., "How Much of the Content in Mathematics Textbooks Is New?" *Mathematics Teacher 35*, (1), pp. 18-23, 1987.

Kieran, C. & Chalouh, L., "Prealgebra: The Transition from Arithmetic to Algebra." In D. T. Owens, *Research Ideas for the Classroom: Middle Grades Mathematics* (New York, NY: Macmillan, 1993).

Kieran, C., "The Learning and Teaching of School Algebra." In D. A. Grouws, *Handbook of Research on Mathematics Teaching and Learning* (New York, NY: Macmillan, 1992).

Table 1.9

Teachers' Reports on the Instructional Emphasis Placed on Algebra and Functions, Grades 4 and 8

		HEAVY EMPHASIS		MODERATE EMPHASIS		LITTLE OR NO EMPHASIS	
	Assessment Years	Percentage of Students	Algebra Proficiency	Percentage of Students	Algebra Proficiency	Percentage of Students	Algebra Proficiency
Grade 4							
Nation	1992	4 (0.9)	220 (3.6)	30 (3.1)>	219 (2.2)	66 (3.2)<	218 (1.1)
	1990	2 (0.7)	207 (8.8)	17 (2.4)	213 (2.9)	82 (2.5)	215 (1.1)
Top One-Third	1992	3 (1.3)	241 (5.8)	37 (7.8)>	237 (2.6)	60 (7.7)<	235 (1.5)>
	1990	2 (1.6)	233 (5.3)	13 (3.7)	230 (2.5)	85 (3.7)	230 (1.6)
Bottom One-Third	1992	3 (0.9)	191 (6.4)	32 (3.2)	195 (2.5)	65 (3.5)	198 (1.7)
	1990	4 (1.7)	192 (5.9)	24 (6.3)	197 (2.6)	72 (6.1)	196 (2.0)
High Ability	1992	7 (3.5)	235(14.5)	28 (8.1)	244 (8.0)	65 (7.4)	235 (2.5)
	1990	1 (1.0)	—	23 (6.1)	224 (6.4)	76 (6.3)	240 (6.2)
Average Ability	1992	2 (0.7)	224 (9.4)	29 (4.6)	223 (2.7)>	69 (4.7)	221 (1.4)
	1990	1 (1.0)	236 (6.8)	18 (4.3)	209 (4.8)	81 (4.3)	216 (1.7)
Low Ability	1992	1 (0.9)	173 (8.4)<	30 (4.7)	194 (5.2)	68 (4.5)	197 (2.2)
	1990	2 (2.5)	228 (4.5)	20 (6.9)	207(13.0)	77 (7.4)	201 (3.0)
Mixed Ability	1992	5 (1.9)	218 (3.1)>	32 (3.9)>	217 (1.9)	62 (4.3)<	217 (1.9)
	1990	3 (1.3)	191 (6.6)	14 (3.9)	216 (3.2)	84 (4.1)	212 (1.8)
Grade 8							
Nation	1992	48 (2.0)	281 (1.6)	40 (2.1)	261 (1.3)>	12 (1.4)	245 (2.5)
	1990	48 (3.5)	276 (2.2)	34 (3.0)	255 (2.0)	18 (2.9)	246 (3.1)
Top One-Third	1992	59 (2.9)	298 (2.2)>	34 (2.9)	278 (1.9)	7 (1.5)	270 (3.9)
	1990	58 (7.0)	289 (2.7)	31 (4.5)	272 (2.6)	11 (3.1)	262 (5.6)
Bottom One-Third	1992	40 (3.7)	259 (1.8)	41 (3.9)	243 (2.1)	18 (2.7)	230 (2.7)
	1990	39 (5.5)	254 (4.2)	32 (5.8)	242 (3.8)	29 (6.5)	234 (4.3)
High Ability	1992	92 (2.4)	301 (1.8)>	7 (2.1)	290 (5.9)	1 (0.6)	248(25.6)
	1990	87 (3.7)	289 (2.7)	11 (3.3)	277 (3.4)	2 (1.3)	297 (3.0)
Average Ability	1992	38 (3.3)	270 (1.9)	52 (3.7)	264 (1.8)	10 (2.2)	255 (4.5)
	1990	39 (5.3)	266 (3.2)	42 (4.9)	258 (3.4)	19 (4.0)	250 (4.5)
Low Ability	1992	18 (3.6)	244 (2.2)	50 (5.7)	248 (2.7)	33 (4.7)	239 (3.3)
	1990	19 (4.5)	254 (7.8)	39 (6.2)	243 (4.0)	42 (6.7)	239 (4.7)
Mixed Ability	1992	45 (4.9)	266 (1.8)	43 (4.5)	263 (2.0)	12 (3.2)	238 (5.2)
	1990	42 (9.1)	266 (3.2)	43 (8.2)	253 (4.2)	15 (5.7)	242 (7.2)

NOTE: In both 1990 and 1992, the question specified informal introduction of concepts at grade 4.

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

— Sample size insufficient to permit a reliable estimate.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Instructional Emphasis on Mathematics Processes

A separate set of four questions at grades 4 and 8 solicited teachers' views about the emphasis they placed on the following specific mathematics skills and abilities:

- learning mathematics facts and concepts
- learning skills and procedures needed to solve problems
- developing reasoning ability to solve unique problems
- learning how to communicate ideas in mathematics effectively

Tables 1.10 through 1.13 provide teachers' responses to these questions for students at both grades 4 and 8. In 1992, 96 percent of grade 4 students received a heavy emphasis on learning facts and concepts, and 92 percent received a heavy emphasis in learning skills and procedures to solve problems. Slightly fewer than half of the fourth graders, 48 percent, received a heavy instructional emphasis in developing reasoning ability related to solving unique problems, and another 44 percent received a moderate level of instructional emphasis. Thirty-eight percent of the fourth graders received a heavy instructional emphasis on learning how to communicate mathematical ideas effectively, and another 50 percent received moderate instructional emphasis.

The percentages of students receiving a heavy instructional emphasis in the areas of facts/concepts and skills/procedures both significantly increased compared to 1990. In contrast, there were no significant increases in the percentages of students receiving heavy or moderate emphasis in reasoning or communication.

For fourth graders in classes of differing ability levels, significant increases in heavy emphasis on learning facts and concepts occurred in the high-ability and average-ability classes. At grade 8, significant increases in heavy emphasis took place for the nation and for students in average- and low-ability classes.

The class ability-level data for fourth graders showed modest, but non-significant, increases in the percentages of students in high-, average-, and low-ability classes receiving heavy emphasis in mathematical skills and procedures. The only significant increases in heavy emphasis came for students at the national level and for students in mixed-ability classes.

At grade 8, teachers reported that about three-fourths of the students were receiving heavy emphasis both in learning mathematics facts and concepts (74 percent) and in the skills and procedures for problem solving (79 percent). These were upward trends compared to 1990, when 55 and 67 percent of the eighth graders, respectively, were receiving heavy instructional emphasis in these two areas.

At grade 8, there also were increases in the percentages of students in average- and low-ability classrooms receiving heavy emphasis in the area of skills and procedures.

About half of the eighth graders were receiving heavy instructional emphasis in developing the reasoning ability to solve unique problems and 40 percent were receiving heavy instructional emphasis in communicating mathematics ideas effectively.

Teachers reported no increases between 1990 and 1992 in the percentages of students receiving heavy emphasis in reasoning and communication for either grade 4 or grade 8, although the percentages of eighth graders in low-ability classes receiving a moderate level of instructional emphasis did increase.

The overall picture of emphasis in the curriculum on facts and concepts, skills and procedures, reasoning, and communication changed between 1990 and 1992 in positive directions, but the overall percentages and the relative changes are not entirely consonant with the recommendations for reform in school mathematics.[17] The data for grades 4 and 8 reflect significant changes in bolstering the traditional strongholds of facts and concepts, and skills and procedures. On the other hand, the process areas of reasoning and communication in problem-solving situations did not receive significantly stronger emphases between 1990 and 1992. When one compares the overall percentages of students receiving heavy emphases in the latter two areas compared with the former two, the lack of emphasis on higher-level processes becomes evident.

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[17] O'Daffer, P. G. & Thornquist, B. A., "Critical Thinking, Mathematical Reasoning, and Proof." In P. S. Wilson, *Research Ideas for the Classroom: High School Mathematics* (New York, NY: Macmillan, 1993).

National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 1989).

Table 1.10

Teachers' Reports on the Instructional Emphasis Placed on Learning Mathematics Facts and Concepts, Grades 4 and 8

	Assessment Years	HEAVY EMPHASIS		MODERATE EMPHASIS		LITTLE OR NO EMPHASIS	
		Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Grade 4							
Nation	1992	96 (0.7)>	218 (1.0)>	4 (0.7)<	216 (3.4)	0 (0.0)	—
	1990	88 (2.4)	214 (1.1)	11 (2.4)	219 (3.9)	0 (0.3)	—
Top One-Third	1992	95 (1.4)	236 (1.1)>	5 (1.4)	228 (3.0)	0 (0.0)	—
	1990	84 (5.8)	228 (1.8)	15 (5.8)	233 (5.9)	1 (0.9)	274(29.1)
Bottom One-Third	1992	96 (1.2)>	197 (1.5)	4 (1.2)<	191 (5.4)	0 (0.0)	—
	1990	86 (2.8)	195 (1.8)	14 (2.8)	195 (4.2)	0 (0.0)	—
High Ability	1992	97 (1.7)>	238 (2.5)	3 (1.7)<	209(21.1)	0 (0.0)	—
	1990	76 (7.2)	232 (5.1)	21 (6.1)	243 (8.2)	3 (3.3)	—
Average Ability	1992	97 (0.9)>	222 (1.3)>	3 (0.9)<	211 (6.5)	0 (0.0)	—
	1990	90 (2.5)	214 (1.8)	10 (2.5)	216 (5.0)	0 (0.3)	—
Low Ability	1992	94 (2.4)	195 (2.2)	6 (2.4)	206 (8.4)	0 (0.0)	—
	1990	88 (4.1)	204 (3.8)	12 (4.4)	190 (9.0)	0 (0.0)	—
Mixed Ability	1992	95 (1.7)	216 (1.3)	5 (1.5)	224 (3.7)	0 (0.0)	—
	1990	89 (5.8)	212 (1.4)	11 (5.8)	218(11.2)	0 (0.0)	—
Grade 8							
Nation	1992	74 (1.8)>	267 (1.1)	23 (1.9)<	271 (2.3)>	3 (0.9)	283 (3.2)>
	1990	55 (3.9)	264 (2.0)	38 (4.0)	262 (2.3)	7 (1.6)	265 (4.8)
Top One-Third	1992	70 (4.5)	288 (2.1)	26 (4.5)	290 (2.4)>	4 (1.1)	293 (5.1)
	1990	57 (5.9)	285 (2.7)	38 (5.9)	274 (1.7)	5 (3.1)	274(12.3)
Bottom One-Third	1992	79 (3.4)>	246 (1.4)	19 (3.1)<	246 (2.9)	2 (0.8)	280 (8.8)
	1990	54 (7.2)	239 (3.3)	40 (7.7)	250 (3.5)	6 (2.4)	255 (9.3)
High Ability	1992	72 (3.4)	299 (2.0)	22 (3.2)	299 (3.0)>	6 (1.3)	301 (4.5)
	1990	60 (4.7)	292 (3.2)	33 (4.5)	283 (5.0)	7 (2.5)	285 (5.4)
Average Ability	1992	73 (3.4)>	265 (1.4)	24 (3.9)<	267 (2.8)	3 (1.0)	274 (4.1)
	1990	47 (5.8)	260 (3.7)	45 (5.6)	261 (2.8)	8 (3.0)	257 (5.8)
Low Ability	1992	83 (2.8)>	244 (1.8)	17 (2.8)<	245 (6.0)	0 (0.4)	—
	1990	59 (6.1)	243 (4.6)	36 (6.7)	245 (4.2)	5 (3.0)	244 (6.2)
Mixed Ability	1992	70 (5.8)	259 (1.8)	25 (4.8)	266 (3.4)	5 (2.3)	270 (5.7)
	1990	61 (7.5)	257 (3.2)	33 (7.8)	255 (7.1)	6 (3.3)	266 (8.7)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). When the proportion of students in either 0 percent or 100 percent, the standard error is inestimable. However, percentages 99.5 percent and greater were rounded to 100 percent and percentages 0.5 percent or less were rounded to 0 percent. Percentages may not total 100 percent due to rounding error.

— Sample size insufficient to permit a reliable estimate.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 1.11

Teachers' Reports on the Instructional Emphasis Placed on Learning Skills and Procedures Needed to Solve Problems, Grades 4 and 8

		HEAVY EMPHASIS		MODERATE EMPHASIS		LITTLE OR NO EMPHASIS	
	Assessment Years	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Grade 4							
Nation	1992	92 (1.0)>	218 (1.0)>	8 (1.0)<	216 (2.2)	0 (0.1)	—
	1990	85 (2.3)	214 (1.2)	15 (2.3)	216 (3.0)	0 (0.1)	—
Top One-Third	1992	91 (2.0)	236 (1.1)>	9 (2.0)	230 (2.4)	0 (0.0)	—
	1990	82 (4.9)	230 (1.8)	18 (4.9)	228 (4.5)	0 (0.3)	—
Bottom One-Third	1992	93 (1.6)	197 (1.5)	7 (1.6)	191 (3.2)	0 (0.0)	—
	1990	86 (2.7)	195 (1.8)	13 (2.8)	196 (4.1)	0 (0.1)	—
High Ability	1992	90 (4.1)	240 (3.1)	10 (4.1)	221 (7.0)	0 (0.0)	—
	1990	86 (6.5)	234 (4.8)	14 (6.5)	251(13.1)	0 (0.0)	—
Average Ability	1992	91 (1.9)	222 (1.3)>	9 (1.9)	220 (3.7)	0 (0.2)	—
	1990	86 (3.2)	214 (1.8)	13 (3.1)	217 (4.0)	0 (0.3)	—
Low Ability	1992	93 (2.3)	196 (2.2)	7 (2.3)	197 (6.0)	0 (0.0)	—
	1990	85 (5.9)	203 (4.1)	15 (5.9)	195 (8.4)	0 (0.3)	—
Mixed Ability	1992	95 (1.3)>	217 (1.3)	5 (1.3)<	214 (4.3)	0 (0.0)	—
	1990	83 (4.6)	212 (1.9)	17 (4.6)	214 (3.6)	0 (0.0)	—
Grade 8							
Nation	1992	79 (1.8)>	268 (1.2)	19 (1.7)<	270 (2.5)	2 (0.5)	287(10.2)
	1990	67 (3.7)	264 (1.8)	29 (3.4)	261 (2.4)	4 (1.2)	272 (6.7)
Top One-Third	1992	78 (4.2)	287 (1.4)>	20 (3.9)	292 (3.6)>	2 (1.1)	316(11.8)
	1990	72 (6.5)	280 (1.9)	26 (6.4)	280 (3.0)	3 (2.2)	291 (4.1)
Bottom One-Third	1992	78 (2.7)	246 (1.5)	21 (2.6)	248 (2.4)	1 (0.7)	233 (8.3)
	1990	68 (5.5)	244 (2.7)	29 (4.5)	244 (3.9)	3 (1.6)	248(10.0)
High Ability	1992	75 (4.2)	299 (1.8)>	21 (3.8)	299 (3.4)>	4 (1.5)	312 (6.8)>
	1990	70 (5.7)	291 (2.5)	24 (5.1)	280 (6.1)	5 (2.2)	289 (5.1)
Average Ability	1992	81 (2.2)>	266 (1.4)	18 (2.0)	265 (2.5)	1 (0.7)	274 (1.7)
	1990	66 (6.0)	262 (2.7)	30 (5.5)	256 (2.7)	4 (1.9)	260(11.7)
Low Ability	1992	84 (2.3)>	245 (2.2)	15 (2.3)	240 (3.5)	0 (0.4)	—
	1990	70 (4.9)	243 (3.7)	28 (5.2)	246 (5.4)	2 (1.7)	256 (5.1)
Mixed Ability	1992	76 (4.0)	261 (2.1)	22 (3.6)	264 (3.3)	2 (1.2)	246(15.1)
	1990	69 (7.1)	254 (3.4)	28 (7.1)	262 (6.3)	4 (2.8)	273(11.0)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). When the proportion of students in either 0 percent or 100 percent, the standard error is inestimable. However, percentages 99.5 percent and greater were rounded to 100 percent and percentages 0.5 percent or less were rounded to 0 percent. Percentages may not total 100 percent due to rounding error.

— Sample size insufficient to permit a reliable estimate.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 1.12

Teachers' Reports on the Instructional Emphasis Placed on Developing Reasoning Ability to Solve Unique Problems, Grades 4 and 8

		HEAVY EMPHASIS		MODERATE EMPHASIS		LITTLE OR NO EMPHASIS	
	Assessment Years	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Grade 4							
Nation	1992	48 (2.9)	220 (1.5)>	44 (2.9)	217 (1.3)	7 (1.3)	215 (2.5)
	1990	44 (3.9)	213 (1.9)	43 (3.1)	215 (1.5)	12 (2.2)	218 (3.3)
Top One-Third	1992	54 (5.0)	238 (1.8)>	41 (5.3)	233 (1.4)	6 (1.6)	230 (2.2)
	1990	42 (6.5)	228 (2.6)	42 (5.2)	231 (2.2)	16 (4.2)	229 (4.8)
Bottom One-Third	1992	51 (5.0)	198 (2.0)	40 (4.0)	194 (2.3)	9 (2.3)	198 (3.7)
	1990	49 (5.1)	195 (2.6)	38 (3.7)	195 (2.8)	14 (3.3)	198 (2.5)
High Ability	1992	48 (7.5)	242 (5.2)	51 (7.6)	234 (2.7)	1 (0.9)	226 (3.1)
	1990	66 (8.2)	234 (5.4)	30 (8.0)	242 (8.5)	3 (1.3)	220(12.6)
Average Ability	1992	46 (4.9)	223(92.0)>	47 (4.8)	221 (1.7)	6 (1.7)	217 (4.8)
	1990	37 (5.0)	210 (3.0)	51 (4.7)	216 (2.3)	12 (3.4)	227 (5.4)
Low Ability	1992	58 (4.5)	199 (3.3)	38 (4.7)	191 (3.2)	4 (2.0)	198(22.1)
	1990	56 (8.2)	206 (5.9)	33 (7.3)	197 (5.3)	11 (4.3)	200 (4.0)
Mixed Ability	1992	48 (3.7)	219 (1.7)	42 (3.5)	215 (2.1)	10 (2.3)	215 (2.4)
	1990	44 (6.5)	211 (3.4)	43 (5.7)	214 (2.1)	13 (4.0)	214 (4.7)
Grade 8							
Nation	1992	49 (2.1)	275 (1.5)	46 (1.9)	264 (1.4)	5 (0.9)<	249 (3.5)
	1990	46 (3.2)	270 (2.5)	39 (3.2)	260 (2.5)	16 (2.3)	252 (3.1)
Top One-Third	1992	54 (3.6)	293 (2.8)	43 (3.3)	285 (1.6)>	4 (1.2)	272 (4.1)
	1990	50 (4.4)	228 (2.5)	37 (5.8)	275 (3.1)	13 (3.9)	267 (5.2)
Bottom One-Third	1992	43 (3.5)	250 (2.4)	48 (3.7)	245 (2.1)	10 (2.3)	240 (4.0)
	1990	47 (7.0)	250 (3.9)	33 (7.3)	241 (4.0)	20 (4.9)	236 (3.5)
High Ability	1992	72 (3.7)	302 (2.1)>	26 (3.7)	294 (3.0)	1 (0.6)	288 (7.7)
	1990	77 (3.8)	288 (3.0)	20 (3.5)	289 (4.3)	4 (1.4)	282 (7.0)
Average Ability	1992	50 (3.8)	267 (1.6)	46 (3.9)	264 (1.8)	4 (0.6)	258 (4.2)
	1990	42 (4.2)	264 (2.6)	39 (3.6)	259 (3.3)	20 (4.0)	254 (5.1)
Low Ability	1992	28 (4.5)	242 (3.3)	59 (5.0)>	245 (3.0)	13 (2.0)<	244 (6.4)
	1990	32 (6.1)	238 (5.6)	35 (6.3)	244 (5.2)	33 (5.9)	251 (3.8)
Mixed Ability	1992	38 (4.4)	260 (2.4)	55 (4.1)	265 (2.0)	7 (2.6)	240 (6.2)
	1990	33 (6.7)	258 (4.6)	56 (8.1)	259 (4.2)	11 (4.5)	242 (8.1)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 1.13

Teachers' Reports on the Instructional Emphasis Placed on Learning How to Communicate Ideas in Mathematics Effectively, Grades 4 and 8

		HEAVY EMPHASIS		MODERATE EMPHASIS		LITTLE OR NO EMPHASIS	
	Assessment Years	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Grade 4							
Nation	1992	38 (2.4)	219 (2.0)>	50 (2.2)	217 (1.2)	12 (1.4)	219 (1.8)
	1990	40 (3.7)	212 (1.9)	43 (3.2)	216 (1.7)	17 (3.0)	217 (2.2)
Top One-Third	1992	40 (4.2)	239 (2.2)	47 (3.6)	235 (1.1)	13 (2.9)	231 (2.4)
	1990	27 (4.7)	229 (3.5)	49 (5.5)	231 (2.2)	24 (5.2)	226 (2.1)
Bottom One-Third	1992	46 (4.1)	198 (1.6)	47 (3.4)	195 (2.4)	7 (1.8)	200 (5.4)
	1990	45 (6.0)	195 (2.8)	40 (4.6)	195 (2.9)	15 (4.4)	196 (3.5)
High Ability	1992	35 (7.2)	246 (6.9)	60 (8.5)	234 (2.5)	4 (2.5)	228 (5.7)
	1990	56 (9.9)	232 (5.9)	37 (9.0)	241 (6.5)	7 (4.3)	247(29.5)
Average Ability	1992	37 (4.0)	222 (2.5)>	49 (4.4)	221 (1.5)	14 (2.3)	222 (2.8)
	1990	31 (4.9)	207 (3.0)	50 (4.7)	219 (2.6)	19 (4.4)	217 (2.9)
Low Ability	1992	36 (5.2)	194 (2.6)	51 (5.5)	194 (3.1)	14 (4.5)	207 (8.2)
	1990	55 (8.6)	206 (5.5)	40 (8.2)	196 (3.9)	4 (2.5)	210(19.4)
Mixed Ability	1992	39 (3.5)	218 (2.2)>	50 (3.3)	216 (1.7)	11 (3.1)	218 (2.8)
	1990	39 (5.8)	209 (2.2)	40 (6.0)	213 (3.2)	21 (5.0)	217 (2.7)
Grade 8							
Nation	1992	40 (2.3)	273 (1.6)>	50 (2.5)	266 (1.2)	10 (1.7)<	266 (3.9)
	1990	38 (3.4)	265 (2.8)	42 (3.5)	265 (2.1)	20 (2.8)	259 (2.9)
Top One-Third	1992	49 (4.1)	290 (2.9)	40 (4.5)	288 (1.8)	11 (3.0)	288 (2.9)>
	1990	38 (5.2)	286 (2.3)	42 (7.0)	280 (3.0)	20 (5.0)	272 (2.9)
Bottom One-Third	1992	36 (3.7)	246 (2.2)	54 (4.2)	248 (1.7)	9 (2.7)	240 (5.5)
	1990	54 (6.2)	247 (3.5)	30 (4.9)	245 (5.2)	17 (3.6)	234 (5.1)
High Ability	1992	52 (3.3)	302 (2.5)>	40 (3.3)	298 (2.5)	8 (2.7)	294 (3.1)
	1990	52 (5.3)	287 (3.9)	41 (5.5)	292 (2.7)	8 (2.7)	291 (4.7)
Average Ability	1992	41 (4.1)	266 (2.1)	52 (4.3)	265 (1.7)	7 (2.1)<	263 (7.8)
	1990	34 (5.0)	262 (3.4)	44 (3.8)	259 (3.4)	22 (4.3)	260 (5.0)
Low Ability	1992	27 (4.6)	241 (2.4)	56 (4.5)>	244 (2.2)	17 (4.9)	251 (5.1)
	1990	35 (6.1)	241 (5.0)	31 (5.1)	249 (4.9)	33 (5.9)	242 (4.3)
Mixed Ability	1992	34 (4.4)	259 (2.9)	56 (4.8)	262 (2.2)	11 (2.3)	265 (4.6)
	1990	33 (7.1)	252 (5.4)	45 (8.8)	257 (4.2)	22 (6.2)	263 (6.9)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Summary

In 1992, 73 percent of the fourth-grade students and 67 percent of the eighth-grade students were in schools that placed a special priority on mathematics. This represented no change from 1990. At grade 4, schools placed about the same emphasis on mathematics as they did on writing, with reading receiving greater priority. At grade 8, reading, writing, and mathematics received about the same emphasis. The priority given all three of these fundamental curriculum areas for eighth graders was higher in bottom one-third schools than in top one-third schools.

At grade 4, 71 percent of the students received at least four hours of mathematics instruction per week. However, at grade 8, only 32 percent of the students received at least four hours of instruction per week. There was no change in teachers' reports about the amounts of homework assigned fourth graders between 1990 and 1992. Teachers reported that the majority of fourth graders (52 percent) were assigned 15 minutes of mathematics homework each day. At grade 8, teachers reported assigning more mathematics homework in 1992 than in 1990, as fewer students were being given only 15 minutes of mathematics homework per day (28 compared to 41 percent). In the top one-third of the schools, 25 percent of the students were expected to do 45 minutes or more of homework per day compared to only 16 percent of the students in the lower one-third of the schools. Students reported homework trends similar to those that their teachers reported, with an increase at grade 8. Twelfth graders enrolled in mathematics classes reported doing more homework than fourth and eighth graders, but the results were comparable to those obtained in 1990.

At grade 4, the mathematics curriculum was heavily focused in the content area of numbers and operations. Teachers reported that 90 percent of the fourth graders were receiving heavy instructional emphasis in this area. Although few (13 percent) were receiving heavy instructional emphasis in measurement, 81 percent were receiving a moderate emphasis, which was up from 70 percent in 1990. Similarly, few were receiving heavy instructional emphasis in geometry, but 71 percent were receiving moderate emphasis, also an increase compared to the 58 percent in 1990. The majority (54 percent) were receiving little or no emphasis in data analysis or introductory algebra concepts (66 percent). In 1990, however, 82 percent of the fourth graders received little or no emphasis in introductory algebra concepts.

Seventy-six percent of the eighth graders were receiving heavy instructional emphasis in numbers and operations. Far fewer were receiving heavy instructional emphasis in measurement (16 percent), geometry (18 percent), or data analysis (11 percent). The percentages receiving moderate emphasis in measurement and geometry did increase substantially between 1990 and 1992, from 50 to 69 percent and from 51 to 71 percent, respectively. Almost half (48 percent) were receiving heavy emphasis in algebra and functions, which represented no difference from 1990.

Teachers' reports did indicate some movement toward a greater balance across the content areas since more fourth and eighth graders were receiving at least a moderate emphasis in measurement and geometry. At grade 4, more students also were receiving some introduction to algebra concepts.

Across the process areas, 96 percent of the fourth graders received heavy emphasis on learning facts and concepts and 92 percent on learning skills and procedures to solve problems. Both of these results were increases compared to 1990. Fewer than half the fourth graders were receiving heavy instructional emphasis in developing reasoning ability related to solving unique problems (48 percent) or in learning how to communicate mathematical ideas effectively (38 percent).

At grade 8, 74 percent were receiving heavy emphasis on learning facts and concepts, and 79 percent were receiving heavy emphasis in skills and procedures for problem solving. Similar to the findings at grade 4, these figures represented significant increases compared to 1990. Forty-nine percent were receiving heavy instructional emphasis in developing reasoning ability related to solving unique problems and 40 percent were receiving such emphasis on learning how to communicate mathematical ideas effectively.

Teachers reported no increases between 1990 and 1992 in the percentages of students receiving heavy emphasis in reasoning and communication for either grade 4 or grade 8.

2

Mathematics Teachers and Their Classes

While priorities in the curriculum and the expectations conveyed to students by homework and time allocations for mathematics instruction are crucial contextual features to learning, there are other important elements as well. Factors such as teachers' training and the types of resources available to them can influence mathematics proficiency. This chapter provides information on teachers' background, class size, ability grouping, and resource availability.

Teachers' Experience

The quality of the teaching staff forms another important characteristic of schools. The extent of their classroom experience as well as the education teachers undergo both before they begin teaching and as part of staff development activities can help to shape their teaching methods and ideas.[18]

As shown in Table 2.1, mathematics teachers reported considerable overall experience, which reflected no change from 1990. In 1992, at both grades 4 and 8, approximately 35 to 36 percent of the students were taught by individuals with 10 years or less experience, 46 to 47 percent by teachers with from 10 to 25 years experience, and 18 percent by teachers with 25 years or more experience. There was some indication, however, that the teachers in top one-third schools were more experienced than those in bottom-performing one-third schools. For example, at grade 8, 28 percent of the students in top one-third schools were taught by teachers with 25 or more years of experience compared to 12 percent of the students in the bottom one-third schools. Teachers may choose to transfer to higher-achieving schools as they gain more teaching experience, or turnover may be lower in the better-performing schools.[19]

At grade 8, teachers also were asked specifically about their years of experience in teaching mathematics. Again, those in the top one-third schools had more experience than those in the bottom one-third of the schools.

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[18] *America's Teachers: Profile of a Profession* (National Center for Education Statistics, U.S. Department of Education, 1993).

McDonnell, L. M., *The Dilemma of Teacher Policy* (Santa Monica, CA: The Rand Corporation, 1989).

[19] Secada, W. G., "Diversity, Equity, and Cognitivist Research." In E. Fennema, T. P. Carpenter, & S. J. Lamon, *Integrating Research on Teaching and Learning Mathematics* (Madison, WI: Wisconsin Center for Education Research, 1988).

Table 2.1

Teachers' Reports on Number of Years' Overall Teaching Experience at the Elementary or Secondary Level, Grades 4 and 8, and Experience Teaching Mathematics, Grade 8

		10 YEARS OR LESS		MORE THAN 10 YEARS BUT LESS THAN 25 YEARS		25 YEARS OR MORE	
	Assessment Years	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Overall Teaching Experience							
Grade 4							
Nation	1992	36 (2.2)	215 (1.5)	46 (2.3)	221 (1.2)	18 (1.4)	221 (1.9)
	1990	34 (2.6)	212 (2.0)	52 (2.9)	214 (1.5)	14 (2.0)	217 (3.1)
Top One-Third	1992	33 (3.3)	234 (1.6)	50 (4.3)	238 (1.4)	17 (2.9)	237 (1.7)
	1990	28 (4.3)	229 (2.2)	55 (5.5)	229 (2.0)	18 (4.0)	231 (4.3)
Bottom One-Third	1992	45 (3.7)	196 (1.7)	39 (3.2)	198 (2.2)	16 (2.7)	200 (1.7)
	1990	40 (4.6)	194 (2.2)	46 (4.7)	196 (2.7)	14 (3.9)	193 (4.8)
Grade 8							
Nation	1992	35 (1.7)	265 (1.7)	47 (1.9)	269 (1.3)	18 (1.6)	275 (2.2)
	1990	31 (3.3)	262 (2.7)	50 (3.5)	265 (1.6)	19 (3.5)	265 (4.1)
Top One-Third	1992	31 (3.7)	287 (2.7)	41 (3.3)	290 (1.9)	28 (3.3)	291 (1.9)
	1990	26 (5.2)	280 (2.5)	54 (7.4)	282 (1.5)	20 (7.6)	281 (5.0)
Bottom One-Third	1992	41 (3.2)	246 (2.1)	47 (3.7)	247 (1.9)	12 (2.0)	244 (3.7)
	1990	35 (6.9)	245 (3.9)	45 (6.1)	245 (2.7)	20 (5.6)	243 (6.6)
Experience Teaching Mathematics							
Grade 8							
Nation	1992	44 (1.9)	265 (1.4)	43 (2.2)	271 (1.5)	13 (1.7)	276 (2.9)
	1990	41 (3.3)	263 (2.3)	45 (3.8)	265 (1.9)	14 (3.3)	266 (5.6)
Top One-Third	1992	37 (3.9)	287 (2.4)	42 (4.6)	291 (1.7)	21 (4.4)	289 (2.5)
	1990	38 (5.4)	282 (2.1)	48 (7.0)	280 (2.0)	14 (7.1)	282 (6.2)
Bottom One-Third	1992	51 (3.1)	246 (1.6)	41 (3.5)	247 (2.2)	8 (1.7)	244 (4.0)
	1990	46 (7.4)	245 (3.0)	40 (6.5)	245 (3.9)	14 (4.9)	242 (9.1)

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Teachers' Undergraduate and Graduate Majors

Many school reform proposals of the 1980s centered on the balance between teachers' coursework in education and their academic field.[20] One popular stance arising from this debate was that elementary school teachers should shift their educational focus more to academics, while secondary school teachers should move in the opposite direction, towards more coursework in the field of education.[21]

Teachers' responses to questions about their undergraduate and graduate majors are presented in Table 2.2. About four-fifths of the fourth graders had mathematics teachers who were education majors, both as undergraduates and graduates. This did not vary much for the top one-third of the schools compared to the bottom-performing one-third of the schools.

At grade 8, 43 percent of the students had teachers who were mathematics majors as undergraduates, 15 percent had teachers who were mathematics education majors, 29 percent had teachers who were education majors, and 13 percent had teachers who majored in other fields. In the top one-third schools, 67 percent of the eighth-grade students had teachers with an undergraduate major in mathematics or mathematics education, but less than half (46 percent) of the students in bottom-performing one-third schools had teachers with a similar mathematics background.

Eighth graders' teachers often switched to an education major when embarking on their graduate study. Approximately half the eighth graders had teachers whose graduate major was education, about one-fifth had teachers whose major was mathematics, about one-fifth had teachers whose major was mathematics education, and the remainder had teachers with no graduate training or other majors. Again, although the differences were not statistically significant, apparently greater percentages of eighth graders in top one-third schools than in bottom-performing one-third schools had teachers with graduate majors in mathematics or mathematics education.

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[20] *America's Teachers: Profile of a Profession* (National Center for Education Statistics, U.S. Department of Education, 1993).

[21] Task Force on Teaching as a Profession, *A Nation Prepared: Teachers for the 21st Century* (New York, NY: Carnegie Forum on Education and the Economy, 1986).

Table 2.2

Teachers' Reports on Their Undergraduate and Graduate Majors, Grades 4 and 8

	MATHEMATICS		MATHEMATICS EDUCATION		EDUCATION		OTHER	
Assessment Year – 1992	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
UNDERGRADUATE MAJOR								
Grade 4								
Nation	6 (1.0)	219 (4.1)	2 (0.5)	231 (6.2)	81 (1.4)	219 (0.9)	12 (1.2)	217 (2.0)
Top One-Third	5 (1.6)	239 (5.6)	2 (0.9)	240 (7.4)	83 (3.1)	236 (0.9)	11 (2.5)	236 (2.8)
Bottom One-Third	7 (2.1)	197 (4.2)	1 (0.4)	192 (8.1)	76 (2.9)	197 (1.4)	16 (2.1)	199 (3.0)
Grade 8								
Nation	43 (2.6)	274 (1.6)	15 (1.8)	270 (2.4)	29 (2.5)	263 (2.0)	13 (1.2)	262 (2.6)
Top One-Third	52 (5.7)	291 (1.9)	15 (4.2)	287 (3.1)	23 (5.5)	286 (2.2)	10 (2.3)	288 (3.5)
Bottom One-Third	32 (3.8)	248 (1.8)	14 (2.4)	248 (3.2)	34 (4.5)	243 (2.7)	19 (2.5)	245 (2.4)

	MATHEMATICS		MATHEMATICS EDUCATION		EDUCATION		OTHER OR NO GRADUATE STUDY	
GRADUATE MAJOR								
Grade 4								
Nation	2 (0.8)	220 (5.7)	4 (0.9)	226 (6.2)	81 (2.1)	220 (1.0)	13 (1.4)	217 (2.3)
Top One-Third	4 (1.7)	233 (3.3)	5 (2.0)	244 (5.9)	78 (3.4)	237 (1.0)	13 (2.6)	231 (2.2)
Bottom One-Third	2 (1.2)	194(10.5)	3 (1.5)	196(12.1)	80 (3.2)	197 (1.6)	15 (2.3)	199 (2.2)
Grade 8								
Nation	21 (2.4)	273 (3.0)	19 (2.1)	274 (2.9)	47 (3.6)	268 (2.0)	13 (1.7)	264 (3.3)
Top One-Third	20 (3.6)	294 (3.8)	24 (3.9)	292 (3.1)	47 (6.0)	287 (1.8)	10 (2.5)	291 (2.9)
Bottom One-Third	19 (4.1)	248 (3.3)	12 (2.6)	248 (4.2)	52 (5.5)	247 (2.3)	18 (4.3)	241 (3.2)

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percents may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematic Assessment

Teachers' Workshops and Training

In addition to their college training, teachers were asked about the amount of time they spent on in-service education in mathematics or the teaching of mathematics during the last year. The results can be found in Table 2.3.

In 1992, 21 percent of the fourth graders had teachers who reported 16 hours or more of school-related professional development activity in mathematics during the past year, which represented an increase from 11 percent in 1990. In 1992, 18 percent of the fourth graders had teachers who reported no in-service training related to mathematics during the past year, which decreased from 32 percent in 1990.

At grade 8, 45 percent of the students had teachers with 16 hours or more of in-service training, 46 percent had teachers with one to 15 hours, and 9 percent had teachers with no such training. These results were essentially the same as in 1990.

There was no consistent relationship between average proficiency and amount of in-service education. At both grades 4 and 8, the amount of in-service training reported by teachers was comparable for top one-third schools and bottom-performing one-third schools.

Table 2.3

Teachers' Reports on the Amount of Time Spent on In-Service Education in Mathematics or the Teaching of Mathematics During the Last Year, Grades 4 and 8

		16 HOURS OR MORE		ONE TO 15 HOURS		NONE	
	Assessment Years	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Grade 4							
Nation	1992	21 (2.3)>	217 (1.7)	62 (2.4)	220 (1.1)>	18 (1.8)<	217 (1.7)
	1990	11 (1.7)	211 (3.8)	57 (3.1)	212 (1.6)	32 (3.0)	217 (2.2)
Top One-Third	1992	21 (3.5)>	234 (1.3)	64 (3.3)	238 (1.1)>	15 (2.9)<	233 (1.9)
	1990	8 (2.6)	227 (3.3)	58 (5.7)	228 (1.9)	34 (5.3)	232 (3.0)
Bottom One-Third	1992	20 (2.5)	192 (2.7)	63 (3.9)	198 (1.6)	17 (2.7)	200 (2.0)
	1990	13 (3.1)	191 (4.5)	63 (4.4)	195 (2.2)	25 (4.0)	198 (2.5)
Grade 8							
Nation	1992	45 (2.5)	269 (1.5)	46 (2.2)	268 (1.3)>	9 (1.4)	268 (3.1)
	1990	37 (3.6)	268 (2.1)	52 (3.8)	261 (1.6)	11 (2.1)	265 (4.0)
Top One-Third	1992	47 (4.4)	290 (2.2)	40 (3.5)	289 (2.1)>	13 (3.6)	284 (1.9)
	1990	37 (5.7)	285 (1.9)	50 (5.8)	279 (1.6)	14 (4.3)	281 (3.8)
Bottom One-Third	1992	43 (3.5)	245 (1.7)	49 (3.6)	247 (1.5)	8 (1.6)	242 (4.4)
	1990	30 (6.2)	253 (5.1)	59 (7.6)	242 (2.4)	11 (4.7)	235 (6.5)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Class Size

Table 2.4 provides information on student proficiency in mathematics by average class size for the nation at grades 4 and 8. No significant differences in students' mathematics proficiency were noted at either grade 4 or 8 across the three class size intervals. Nevertheless, more than 40 percent of the students at both grades were in classes of 26 or more, which may make it difficult for teachers to implement the reasoning and communication goals that require more teacher-pupil contact and assessment than traditional preparation for skill-oriented outcomes. Also, even though statistically significant differences were found only at grade 8, the data at both grades suggest larger class sizes for students in the bottom one-third of the schools than in the top one-third of the schools. This finding corresponds to research on class size showing better student performance and higher affective appreciation for students in smaller classes.[22]

Table 2.4

Teachers' Reports on the Number of Students in Their Mathematics Classes, Grades 4 and 8

	1-20		21-25		26 OR MORE	
Assessment Year – 1992	**Percentage of Students**	**Average Proficiency**	**Percentage of Students**	**Average Proficiency**	**Percentage of Students**	**Average Proficiency**
Grade 4						
Nation	20 (2.6)	220 (2.4)	36 (3.4)	216 (1.6)	43 (4.1)	220 (1.3)
Top One-Third	27 (5.0)	238 (1.7)	29 (6.5)	234 (2.3)	44 (8.6)	236 (2.0)
Bottom One-Third	20 (4.1)	195 (4.1)	38 (4.2)	196 (2.3)	42 (4.9)	199 (2.1)
Grade 8						
Nation	26 (2.0)	266 (1.9)	28 (2.1)	274 (2.5)	46 (2.9)	267 (1.2)
Top One-Third	30 (3.8)	287 (2.1)	33 (3.3)	293 (5.1)	37 (4.3)	287 (1.7)
Bottom One-Third	25 (3.1)	245 (2.9)	22 (3.0)	248 (2.5)	53 (5.1)	248 (1.7)

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix B for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

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[22] Glass, G. V., Cahen, L. S., Smith, M. L., & Filby, N. N., *School Class Size: Research and Policy* (Beverly Hills, CA: Sage Publications, 1992).

Assigning Students to Mathematics Classes by Ability Grouping

Probably no issue in instructional management has been more heavily researched and debated in the past few years than the use of ability grouping. As shown in Table 2.5, there was essentially no difference between 1990 and 1992 in teachers' reports that students were assigned to their mathematics classes by ability groups. In 1992, 25 percent of the fourth graders were assigned to ability-grouped classes for instruction in mathematics as part of school practice. However, by grade 8, teachers reported that 58 percent of the students were in schools where students were assigned to mathematics classes by ability. Data in the Second International Mathematics Study (SIMS) and other studies of mathematics classroom learning have indicated that the indiscriminate application of ability grouping can hinder mathematics students. Such grouping has the potential to limit students' opportunity to learn important mathematics content and processes and track them into quantitatively and qualitatively different streams in the curriculum, some of which do not progress much beyond high school geometry at best.[23]

Others suggest more and earlier ability grouping to section off our best students and offer them the best that our school mathematics curricula can provide in an attempt to maximize the content potential for individual students and help make the U.S. competitive internationally in mathematics and science.[24]

At grade 4, there was no difference in average proficiency between those students who were assigned to their mathematics classes by ability and those who were not. However, at grade 8, the students in assigned ability groups for mathematics instruction had significantly higher average proficiency.

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[23] McKnight, C.C., et al., *The Underachieving Curriculum: Assessing U.S. School Mathematics from an International Perspective* (Champaign, IL: International Association for the Evaluation of Educational Achievement, 1987).

Oakes, J., "Can Tracking Research Inform Practice? Technical, Normative, and Political Considerations," *Educational Researcher*, 21 (No. 4, May), 12-21, 1992.

Secada, W., "Race, Ethnicity, Social Class, Language, and Achievement in Mathematics." In D.A. Grouws, *Handbook of Research on Mathematics Teaching and Learning*, (New York, NY: Macmillan, 1992).

[24] Gallagher, J. E., "Ability Grouping: A Tool for Educational Excellence," *The College Board Review, 168*, Summer 1993, pp. 21-27.

National Excellence: The Case for Developing America's Talent (Office of Educational Research and Improvement, U.S. Department of Education, U.S. Government Printing Office, 1993).

Table 2.5

Teachers' Reports on Students Being Assigned to Mathematics Classes Based on Ability Grouping, Grades 4 and 8

	Assessment Years	YES, STUDENTS GROUPED BY ABILITY Percentage of Students	YES, STUDENTS GROUPED BY ABILITY Average Proficiency	NO, STUDENTS NOT GROUPED BY ABILITY Percentage of Students	NO, STUDENTS NOT GROUPED BY ABILITY Average Proficiency
Grade 4					
Nation	1992	25 (2.6)	221 (2.3)	75 (2.6)	217 (1.0)>
	1990	28 (4.0)	220 (2.5)	72 (4.0)	212 (1.1)
Top One-Third	1992	31 (5.5)	241 (2.3)	69 (5.5)	234 (1.0)>
	1990	34 (8.1)	234 (2.6)	66 (8.1)	228 (1.4)
Bottom One-Third	1992	24 (4.6)	191 (1.9)	76 (4.6)	198 (1.6)
	1990	20 (5.1)	195 (3.7)	80 (5.1)	195 (2.1)
Grade 8					
Nation	1992	58 (2.4)	274 (1.5)	42 (2.4)	261 (1.2)>
	1990	61 (3.8)	269 (2.1)	39 (3.8)	254 (2.3)
Top One-Third	1992	66 (3.7)	293 (1.7)>	34 (3.7)	281 (2.1)>
	1990	72 (8.3)	283 (1.8)	28 (8.3)	273 (2.4)
Bottom One-Third	1992	50 (5.0)	250 (2.1)	50 (5.0)	243 (2.1)>
	1990	54 (7.1)	251 (3.6)	46 (7.1)	236 (2.3)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details).

SOURCE: National Assessment of Educational Progress (NAEP) 1992, Mathematics Assessment

Teachers' Reports on the Ability Levels of Their Students' Classes

While grade 4 teachers reported that only 25 percent of the students were assigned to their mathematics classes by ability as the result of school practice, considerably more ability grouping may actually occur as a result of factors other than school policy, including the socioeconomic standing of the community.[25] The data in Table 2.6, which contains teachers' views of the ability levels of their students' classes, suggest that about 60 percent of the grade 4 students may be, in effect, ability grouped. Similar reports from grade 8 teachers suggest that as many as 79 percent of the eighth graders may be in ability-grouped classes of one type or another.

As might be anticipated, for both grades 4 and 8, larger percentages of students in top one-third schools than in bottom one-third schools were reported as being in high-ability classes and smaller percentages were reported as being in low-ability classes. The converse was reported in bottom one-third schools.

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[25] Ekstrom, R. B., "Six Urban School Districts: Their Middle Grade Mathematics Grouping Policies and Practices." Paper presented at the American Educational Research Association Symposium, *On the Right Track: The Consequences of Mathematics Course Placement Policies and Practices in the Middle Grades* (Chicago, IL: 1991).

Table 2.6

Teachers' Reports on the Ability Levels of Students in Their Mathematics Classes, Grades 4 and 8

		PRIMARILY HIGH ABILITY		PRIMARILY AVERAGE ABILITY		PRIMARILY LOW ABILITY		WIDELY MIXED ABILITY	
	Assessment Years	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Grade 4									
Nation	1992	9 (1.3)	238 (2.5)	42 (2.1)	222 (1.3)>	12 (1.1)	196 (2.0)	38 (2.5)	217 (1.2)
	1990	11 (1.8)	236 (5.1)	36 (2.5)	215 (1.7)	12 (1.8)	202 (3.7)	41 (3.9)	212 (1.6)
Top One-Third	1992	17 (3.8)	244 (3.3)	48 (4.0)>	236 (1.6)>	5 (1.4)	227 (3.4)	30 (4.6)	232 (1.5)>
	1990	15 (3.8)	251 (3.9)	33 (4.2)	227 (2.4)	8 (2.8)	224 (4.2)	43 (6.1)	225 (2.1)
Bottom One-Third	1992	3 (0.8)	210 (5.3)	34 (3.8)	201 (2.1)	21 (2.2)	182 (2.6)	42 (3.7)	200 (1.7)
	1990	7 (2.4)	212 (4.2)	32 (4.4)	194 (3.2)	17 (3.3)	187 (4.5)	44 (7.0)	196 (2.4)
White	1992	10 (1.6)	241 (2.9)	44 (2.5)	227 (1.3)>	9 (1.2)	210 (2.5)	37 (3.1)	224 (1.4)>
	1990	12 (2.2)	240 (5.9)	36 (2.9)	220 (1.9)	10 (1.8)	210 (4.2)	42 (4.4)	218 (1.7)
Black	1992	2 (0.7)	206 (5.4)	36 (3.6)	199 (2.4)	21 (2.6)	176 (2.9)<	42 (3.6)	194 (2.4)
	1990	6 (1.9)	208 (4.6)	32 (4.6)	189 (4.1)	20 (5.0)	191 (4.8)	42 (7.8)	191 (3.5)
Hispanic	1992	7 (1.9)	212 (7.6)	36 (3.3)	206 (2.2)	22 (2.8)	184 (3.1)	36 (3.3)	203 (1.9)
	1990	6 (1.6)	222 (8.6)	44 (4.7)	204 (2.6)	18 (3.3)	184 (5.5)	32 (5.5)	196 (3.0)
Asian/Pacific Islander	1992	21 (5.0)	243 (5.0)	39 (5.8)	235 (6.1)	6 (2.9)	194 (7.7)<	35 (5.1)	230 (4.7)
	1990	22 (5.5)	236 (8.3)	34 (6.7)	228(10.2)	10 (4.3)	234(10.1)	34 (6.7)	218 (7.4)
American Indian	1992	8 (2.7)	202(17.1)	44 (7.8)	213 (4.8)	13 (4.0)	196 (9.9)	35 (7.3)	208 (4.9)
	1990	4 (2.1)	231(16.5)	22 (5.5)	210 (7.2)	16 (6.7)	210(16.7)	57 (8.1)	204 (6.0)
Male	1992	9 (1.5)	240 (3.6)	41 (2.2)	223 (1.6)	12 (1.3)	196 (2.6)	38 (2.7)	218 (1.2)>
	1990	10 (1.9)	239 (5.7)	35 (2.7)	217 (1.9)	14 (2.3)	203 (4.9)	41 (4.2)	212 (2.1)
Female	1992	9 (1.3)	235 (2.7)	42 (2.3)	221 (1.4)>	12 (1.1)	196 (2.2)	37 (2.5)	215 (1.8)
	1990	12 (1.9)	234 (5.6)	37 (2.7)	212 (2.4)	10 (1.6)	200 (3.8)	41 (3.9)	213 (1.9)
Grade 8									
Nation	1992	23 (1.3)	299 (1.8)>	38 (1.9)	265 (1.3)	18 (1.7)	244 (1.9)	21 (2.1)	261 (1.6)
	1990	25 (1.8)	288 (2.3)	34 (2.6)	260 (2.0)	16 (2.1)	244 (3.5)	25 (3.9)	256 (3.1)
Top One-Third	1992	32 (2.7)	310 (2.8)	41 (3.5)	283 (1.6)>	12 (2.2)	263 (4.3)	16 (2.9)	281 (2.0)
	1990	27 (4.4)	303 (2.5)	40 (6.2)	275 (2.6)	11 (3.5)	262 (5.6)	21 (8.6)	274 (2.3)
Bottom One-Third	1992	13 (1.4)<	279 (3.0)	33 (3.5)	244 (2.4)	26 (2.8)	233 (2.9)	27 (4.2)	248 (2.4)
	1990	23 (3.0)	269 (5.2)	30 (3.7)	239 (3.6)	20 (4.2)	229 (5.9)	28 (6.5)	240 (3.3)
White	1992	26 (1.6)	304 (1.8)>	38 (2.1)	274 (1.4)>	15 (1.9)	254 (2.2)	20 (2.2)	269 (1.6)
	1990	26 (2.0)	292 (2.4)	36 (3.2)	266 (2.1)	14 (2.1)	252 (3.7)	24 (4.2)	263 (3.1)
Black	1992	12 (1.8)	261 (5.6)	41 (3.4)	239 (2.1)	27 (4.1)	226 (2.4)	20 (3.2)	236 (3.5)
	1990	18 (3.2)	272 (5.9)	36 (4.2)	235 (4.9)	24 (5.3)	226 (4.9)	21 (6.0)	238 (4.0)
Hispanic	1992	13 (1.4)	280 (4.3)	42 (3.7)	248 (1.9)	25 (2.1)	230 (2.4)	20 (3.9)	244 (3.6)
	1990	20 (2.7)	267 (5.9)	28 (5.9)	246 (3.9)	19 (3.7)	231 (8.5)	34 (7.7)	248 (5.1)
Asian/Pacific Islander	1992	48 (4.8)	308 (4.2)	36 (3.7)>	276 (5.8)	6 (2.2)	254 (9.4)	10 (2.4)	264 (7.1)
	1990	56 (7.0)	294 (5.7)	18 (4.4)	267 (7.8)	14 (5.2)	236(10.8)	10 (7.0)	248(14.2)
American Indian	1992	6 (2.5)	274(12.4)	29 (6.1)	258 (6.1)	19 (4.2)	245 (8.9)	46 (7.3)	253 (3.4)
	1990	4 (5.8)	238(10.7)	25(32.4)	272 (6.5)	8(13.9)	237(15.4)	62(51.1)	237(29.1)
Male	1992	23 (1.2)	299 (2.1)	37 (2.0)	266 (1.7)	20 (1.9)	244 (1.9)	20 (2.0)	262 (2.2)
	1990	24 (1.8)	291 (2.9)	34 (3.0)	262 (2.5)	16 (2.5)	245 (3.7)	26 (4.6)	256 (3.4)
Female	1992	24 (1.5)	299 (2.1)>	39 (1.9)	264 (1.4)>	15 (1.7)	245 (2.5)	22 (2.2)	261 (1.6)
	1990	25 (2.1)	286 (2.3)	35 (2.4)	258 (2.1)	16 (2.0)	242 (4.1)	24 (3.6)	257 (3.5)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Availability of Resources

Table 2.7 presents teachers' reports about the availability of resources for both the 1990 and 1992 assessments. There was essentially no change between assessments in teachers' responses to the question about how well supplied they were by their school systems with the instructional materials and other resources needed to teach their classes. In 1992, teachers of 36 percent of the fourth graders reported receiving only some or none of the resources they needed, while teachers of 11 percent of the students reported that they got all the resources that they needed. The teachers of the remaining 52 percent indicated that they got most of the resources that they needed. Students in classrooms with only some or none of the necessary resources performed significantly lower than the students in classrooms with more resources. Teachers in the bottom one-third performing schools and in disadvantaged urban areas reported less access to materials and resources than teachers in the top one-third schools and in other types of communities.

Table 2.7

Teachers' Reports on the Availability of Resources, Grades 4 and 8

	Assessment Years	I GET SOME OR NONE OF THE RESOURCES I NEED: Percentage of Students	I GET SOME OR NONE OF THE RESOURCES I NEED: Average Proficiency	I GET MOST OF THE RESOURCES I NEED: Percentage of Students	I GET MOST OF THE RESOURCES I NEED: Average Proficiency	I GET ALL THE RESOURCES I NEED: Percentage of Students	I GET ALL THE RESOURCES I NEED: Average Proficiency
How well supplied are you by your school system with the instructional materials and other resources you need to teach your class?							
Grade 4							
Nation	1992	36 (3.1)	214 (1.8)	52 (2.6)	222 (1.1)>	11 (1.5)	222 (2.4)
	1990	37 (2.9)	209 (1.7)	49 (2.8)	217 (1.6)	14 (2.1)	216 (2.6)
Top One-Third	1992	29 (6.2)	234 (1.8)>	58 (4.5)	237 (1.2)	13 (3.2)	238 (2.1)>
	1990	32 (4.4)	224 (1.9)	50 (4.7)	234 (2.2)	18 (4.1)	226 (3.9)
Bottom One-Third	1992	49 (3.8)	196 (1.7)	40 (3.2)	198 (2.1)	11 (2.5)	199 (2.6)
	1990	45 (4.1)	193 (2.2)	40 (4.0)	196 (2.5)	15 (3.8)	200 (3.6)
Advantaged Urban	1992	32 (6.0)	234 (3.9)	57 (5.2)	239 (3.1)	10 (3.5)	245 (2.9)
	1990	14 (7.6)	227 (4.4)	62 (9.5)	232 (4.3)	24 (8.6)	233 (4.6)
Disadvantaged Urban	1992	55 (7.5)	198 (3.1)	37 (7.1)	190 (4.5)	8 (3.1)	188 (5.1)
	1990	52 (8.2)	189 (2.6)	32 (7.2)	198 (6.1)	16 (6.2)	206 (8.9)
Extreme Rural	1992	36 (8.1)	210 (5.0)	50 (6.9)	218 (2.8)	14 (5.9)	221(12.7)
	1990	43 (9.6)	210 (5.7)	50(12.4)	214 (8.3)	7 (5.1)	219 (6.3)
Other	1992	35 (3.4)	215 (2.2)	54 (2.9)	222 (1.1)>	11 (1.7)	221 (2.4)>
	1990	38 (3.9)	212 (1.8)	49 (3.6)	216 (1.7)	13 (2.5)	212 (2.7)
Grade 8							
Nation	1992	33 (1.7)	263 (1.4)	53 (2.2)	271 (1.1)	14 (2.0)	274 (3.1)
	1990	31 (3.9)	260 (2.9)	54 (3.9)	266 (2.0)	15 (2.1)	265 (3.0)
Top One-Third	1992	28 (3.2)	284 (2.5)	53 (3.1)	291 (1.3)>	19 (3.3)	290 (3.1)
	1990	29 (8.2)	280 (3.0)	55 (7.7)	283 (1.7)	16 (4.3)	278 (6.4)
Bottom One-Third	1992	42 (3.0)	244 (1.7)	45 (4.1)	246 (2.1)	12 (3.6)	250 (4.7)
	1990	31 (5.2)	239 (6.0)	59 (5.5)	247 (2.2)	10 (2.9)	242 (7.5)
Advantaged Urban	1992	29 (7.4)	282 (4.6)	47 (7.9)	292 (4.1)	24 (5.3)	289(10.1)
	1990	10 (5.0)	277 (3.5)	54 (8.3)	285 (1.5)	36 (7.4)	274 (7.2)
Disadvantaged Urban	1992	51 (6.6)	238 (3.7)	37 (6.1)	242 (3.8)	12 (4.3)	239 (7.6)
	1990	49(14.0)	251 (5.4)	40(13.1)	255 (6.1)	11 (7.3)	242(10.4)
Extreme Rural	1992	35 (8.2)	265 (6.9)	46(12.0)	272 (5.5)	19(11.5)	262 (3.7)
	1990	43(10.1)	256 (7.4)	55(10.2)	260 (9.5)	2 (2.6)	—
Other	1992	32 (2.4)	264 (1.8)	56 (2.6)	270 (1.2)	12 (1.8)	275 (2.7)>
	1990	31 (5.3)	262 (4.2)	56 (5.1)	265 (2.1)	13 (2.7)	264 (3.1)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

— Sample size insufficient to permit reliable estimate.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Summary

Mathematics teachers at grades 4 and 8 reported having considerable overall experience. In 1992, the majority of both fourth and eighth graders were taught by teachers with more than 10 years teaching experience. More eighth-grade students in top one-third schools than in bottom-performing one-third schools (28 versus 12 percent) were taught by the most experienced teachers, with 25 years or more of experience.

At grade 4, approximately four-fifths of the students had teachers who majored in education. At grade 8, the majority (58 percent) had teachers who majored in mathematics or mathematics education. More students in top one-third schools had teachers who majored in mathematics or mathematics education (67 percent) than did students in bottom-performing one-third schools (46 percent). Eighteen percent of the fourth graders and 9 percent of the eighth graders had teachers who reported no in-service training in mathematics education during the past year, which represented an improvement compared to 1990. These findings did not differ significantly by school performance.

Although there was no clear relationship between class size and average mathematics proficiency, 43 to 46 percent of the fourth and eighth graders were in classes of 26 or more students and larger classes tended to be more prevalent in the bottom-performing one-third of the schools. Research has indicated better student performance and higher affective appreciation for students in smaller classes.

Teachers reported that about one-fourth of the fourth graders and more than half of the eighth graders were assigned to classes by ability for mathematics instruction. At grade 4, there was no difference in proficiency between those assigned to mathematics classes by ability group and those who were not, but at grade 8 the students assigned by ability for mathematics instruction had higher average proficiency. According to teachers' reports about the actual ability levels of their classes, however, approximately 60 percent of the fourth graders and 79 percent of the eighth graders were in classes with students of similar ability (i.e., high-, average-, or low-ability classes). Factors other than school practice, such as the socioeconomic status of the community, also may influence the prevalence of ability grouping.

When asked about resources in general, teachers reported that greater percentages of students in the bottom-performing one-third of the schools than in the top-performing one-third of the schools were in classrooms with only some or none of the necessary resources — 49 compared to 29 percent at grade 4 and 42 compared to 28 percent at grade 8.

3

Instructional Approaches in Mathematics Classes

Instruction in mathematics classrooms across the nation is in a period of transition. As noted in NCTM's *Professional Standards for Teaching Mathematics*, through the 1970s and '80s the most noticeable aspect of mathematics classes was the day-in and day-out routine of correcting the answers to yesterday's problems and having new problems assigned for the next day, usually with a brief explanation.[26]

To achieve the new goals for improved mathematics learning and achievement for our students,[27] however, there need to be accompanying changes in mathematics teaching and instruction. Educational research findings indicate that students learn through actively constructing their own meanings, which means a major shift from accumulating facts and procedures to learning mathematics as an integrated set of intellectual tools for making sense of mathematical situations.[28] Thus, the most effective teaching involves creating mathematical tasks for students, presenting clear expectations of what is to be learned, and engaging them in discourse or written discussion in order to make ongoing instructional decisions.

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[26] Weiss, I., *Science and Mathematics Education Briefing Book* (Chapel Hill, N.C.: Horizon Research, 1989).

Welch, W., "Science Education in Urbanville: A Case Study." In R. Stake and J. Easely, editors, *Case Studies in Science Education* (Urbana, IL: University of Illinois, 1978).

[27] The National Education Goals Report, Building a Nation of Learners (Washington, DC: National Education Goals Panel, Government Printing Office, 1993).

This chapter contains the results to background questions about the materials teachers use and the frequency with which they employ various instructional approaches, including group work, projects, manipulatives, discussion, and writing about problem solving. It also contains information about approaches and frequency of assessment and testing. During this period of reform, it may be that terms no longer mean what they once did. For example, in responding to questions about work in small groups, writing reports or doing mathematics projects, or assessment involving portfolios or presentations, some teachers may have responded from a conventional perspective and others from a reform perspective. Still, given the reports from both students and their teachers about these activities, a picture of school mathematics emerges that suggests some slight movement between 1990 and 1992 toward the type of program suggested by the *NCTM Teaching Standards*, including more group work and less frequent testing. However, progress appears to be slow as classrooms are still heavily reliant on textbooks and activities such as projects and writing about problems are very infrequent.

Use of Instructional Materials: Textbooks and Worksheets

Tables 3.1 and 3.2 present teachers' reports about how often they used printed materials, either textbooks or worksheets, in the course of teaching problem solving. Between 1990 and 1992, teachers' responses showed greater percentages of students at both grades 4 and 8 doing problems from textbooks almost every day — particularly eighth graders in top one-third schools — and greater percentages of eighth graders doing problems from worksheets almost every day — particularly those in bottom one-third schools. Greater percentages of fourth graders in bottom one-third schools also were doing worksheet problems on a daily basis.

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[28] Resnick, L.B., *Education and Learning to Think* (Washington, DC: National Academy Press, 1987).

At grade 4, the trends between 1990 and 1992 have been to increase textbook usage with average- and high-ability students and to increase worksheet use with low-ability students. At grade 8, the pattern is different, in that average- and low-ability students were seeing increased use of the textbook as a source of problems and mixed-ability grade 8 students were seeing an increase in problems from worksheets.

It should be emphasized that both types of materials can be either good or poor sources for problems. Although worksheets have tended to focus on drill and practice, teachers also are developing and sharing innovative materials and activities to supplement their textbooks. Thus, it may be that teachers' reports about increased use of worksheets reflect increased efforts to tailor materials for students' needs. The students, however, had a slightly different perception about the frequency of worksheet use.

Tables 3.3 and 3.4 present the students' reports at grades 4, 8, and 12 on their perceptions about the use of textbooks and worksheets as sources of mathematics problems for classroom work. At grade 4, the students' reports on textbook usage were similar to those made by their teachers. If one combines the "almost every day" and "at least once a week" categories, the same is true for the use of worksheets. However, students' reports for the daily versus weekly categories differed from that of teachers, with the teachers reporting much less reliance on worksheets.

At grade 8, about 85 percent of both teachers and students reported the use of textbooks as a problem source on a daily basis. With worksheets, if one sums the daily and weekly totals, teachers and students were in close agreement. However, if we look at the results for the two individual categories, students reported more daily worksheet usage than teachers.

At grade 12, the only information on the primary source materials for problem solving comes from student reports. Here, 88 percent of the students reported using a textbook daily as a source of problems, while 16 percent reported using worksheets daily for problem-solving work.

Looking at students' reports at grades 4 and 8, those reporting daily or weekly use of the textbook as a source of problem-solving activities performed significantly higher than those reporting less than weekly usage. Eighth graders reporting almost daily usage of worksheets for problem-solving work had significantly lower mathematics proficiency than students reporting usage at least once a week. These students, in turn, performed significantly lower than those students reporting the use of worksheets for problem solving less than weekly. This, in combination with the results showing heavier use of worksheets on a daily basis in bottom one-third schools than in top one-third schools, may indicate that the worksheet usage at grade 8 is primarily for remediation focused on relatively low skills. Research has found that workbooks and worksheets tend to be used for practice and review, and that they are particularly prevalent in remedial programs for low-achieving children.[29] This same pattern between average proficiency and worksheet usage also held true for grade 12 students currently enrolled in mathematics classes.

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[29] Kober, N., *EDTALK, What We Know About Mathematics Teaching and Learning* (Washington, DC: Council for Educational Development and Research, 1991).

Table 3.1

Teachers' Reports on the Frequency with Which Students Do Mathematics Problems from Textbooks in Mathematics Class, Grades 4 and 8

		ALMOST EVERY DAY		AT LEAST ONCE A WEEK		LESS THAN WEEKLY	
	Assessment Years	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Do mathematics problems from textbooks							
Grade 4							
Nation	1992	76 (2.2)>	217 (1.0)>	20 (1.8)	220 (2.6)	4 (1.2)	226 (4.3)
	1990	59 (3.2)	214 (1.0)	37 (3.0)	215 (2.0)	4 (2.2)	223 (7.2)
Top One-Third	1992	72 (5.2)>	234 (1.0)>	22 (4.3)	240 (3.2)	7 (2.8)	236 (5.2)
	1990	52 (5.5)	228 (1.7)	40 (6.1)	231 (2.5)	8 (5.9)	231 (6.3)
Bottom One-Third	1992	80 (2.2)>	197 (1.7)	18 (1.9)<	194 (2.8)	2 (0.9)	192 (6.9)
	1990	60 (5.3)	195 (2.4)	37 (5.0)	195 (2.8)	3 (1.4)	196 (4.7)
High Ability	1992	68 (8.3)>	234 (2.4)	22 (7.2)<	251 (9.6)	10 (7.1)	238(17.5)
	1990	41 (8.1)	226 (5.8)	58 (8.4)	243 (7.2)	2 (1.6)	—
Average Ability	1992	84 (2.7)>	221 (1.2)>	13 (2.0)<	223 (4.1)	3 (1.5)	235 (6.6)
	1990	68 (5.2)	215 (2.0)	32 (5.2)	215 (3.2)	0 (0.0)	—
Low Ability	1992	69 (5.7)	196 (2.4)	28 (5.3)	196 (5.7)	3 (2.5)	188 (7.3)
	1990	48 (8.5)	206 (5.5)	52 (8.5)	198 (4.3)	0 (0.0)	—
Mixed Ability	1992	71 (3.6)	215 (1.6)	25 (3.2)	221 (2.3)>	4 (1.5)	220 (6.5)
	1990	60 (5.8)	213 (1.8)	30 (5.4)	208 (2.5)	10 (5.8)	223 (8.4)
Grade 8							
Nation	1992	83 (1.5)>	272 (1.3)	14 (1.5)<	256 (2.4)	3 (0.6)	250 (6.0)
	1990	64 (3.3)	268 (1.7)	32 (3.1)	255 (2.9)	3 (1.2)	259 (6.5)
Top One-Third	1992	89 (2.7)>	291 (1.7)	10 (2.2)<	272 (2.9)	1 (1.0)	282 (5.2)
	1990	63 (6.5)	286 (2.2)	35 (5.6)	271 (2.4)	1 (1.4)	—
Bottom One-Third	1992	80 (3.1)	250 (1.1)	16 (2.8)<	235 (3.1)	5 (1.1)	228 (6.0)
	1990	63 (6.5)	248 (3.3)	34 (6.9)	235 (4.4)	3 (2.1)	244(12.6)
High Ability	1992	92 (2.3)	300 (1.9)>	5 (1.7)	296 (5.1)	3 (1.2)	282(10.9)
	1990	84 (4.2)	289 (2.5)	15 (3.9)	285 (5.9)	2 (1.5)	286 (6.3)
Average Ability	1992	84 (2.6)>	268 (1.4)	14 (2.5)<	256 (2.9)	2 (0.7)	255 (5.1)
	1990	54 (5.0)	262 (2.8)	43 (4.9)	258 (3.3)	3 (1.9)	260 (3.5)
Low Ability	1992	80 (3.1)>	248 (2.0)	16 (2.6)<	233 (3.8)	4 (1.4)	218 (4.8)
	1990	61 (5.7)	250 (4.2)	37 (5.7)	236 (5.2)	2 (1.3)	218 (3.4)
Mixed Ability	1992	74 (4.4)	262 (1.8)	21 (4.1)	262 (3.4)	5 (2.0)	248(10.1)
	1990	64 (9.4)	258 (2.9)	33 (8.3)	251 (5.9)	2 (1.7)	282(22.5)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). When the proportion of students is either 0 percent or 100 percent, the standard error is inestimable. However, percentages 99.5 percent and greater were rounded to 100 percent and percentages less than 0.5 percent were rounded to 0 percent. Percentages may not total 100 percent due to rounding error.

— Sample size insufficient to permit a reliable estimate.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 3.2

Teachers' Reports on the Frequency with Which Students Do Mathematics Problems from Worksheets in Mathematics Class, Grades 4 and 8

		ALMOST EVERY DAY		AT LEAST ONCE A WEEK		LESS THAN WEEKLY	
	Assessment Years	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Do mathematics problems on worksheets							
Grade 4							
Nation	1992	26 (2.1)	219 (1.6)	56 (2.2)<	218 (1.5)>	18 (1.8)	217 (1.8)
	1990	19 (2.7)	217 (2.6)	68 (2.8)	213 (1.2)	13 (1.6)	218 (2.8)
Top One-Third	1992	27 (4.5)	236 (1.6)>	57 (4.4)	236 (1.6)	16 (2.7)	234 (1.8)
	1990	24 (4.7)	227 (2.7)	59 (5.8)	230 (2.1)	17 (3.5)	231 (2.3)
Bottom One-Third	1992	23 (3.3)>	196 (2.8)	57 (3.5)<	197 (1.8)	20 (3.3)	196 (2.7)
	1990	11 (2.3)	191 (2.4)	77 (3.9)	195 (2.1)	13 (3.3)	196 (4.6)
High Ability	1992	20 (5.2)	227 (5.0)	58 (8.1)	240 (4.0)	21 (5.6)	243 (4.7)
	1990	10 (6.5)	241 (9.0)	68 (7.2)	238 (6.7)	21 (5.3)	228 (6.6)
Average Ability	1992	26 (2.8)	226 (2.4)	57 (3.5)	221 (1.7)>	17 (2.4)	218 (2.2)
	1990	21 (3.3)	219 (2.6)	63 (4.3)	212 (2.3)	16 (3.9)	219 (3.7)
Low Ability	1992	33 (4.3)>	201 (4.1)	52 (6.1)	194 (3.7)	14 (4.3)	191 (3.0)
	1990	11 (5.3)	198 (7.4)	74 (7.9)	201 (3.5)	15 (6.2)	209(16.0)
Mixed Ability	1992	26 (3.9)	218 (2.7)	55 (4.0)	216 (1.4)	18 (3.1)>	217 (3.2)
	1990	22 (4.8)	215 (4.1)	69 (5.0)	211 (2.2)	9 (2.4)	217 (3.5)
Grade 8							
Nation	1992	12 (1.8)>	261 (4.3)	52 (2.2)<	266 (1.5)>	36 (2.5)	275 (1.8)
	1990	6 (1.6)	265 (4.4)	62 (3.4)	258 (1.8)	32 (3.5)	274 (2.5)
Top One-Third	1992	11 (3.4)	282 (7.6)	47 (4.5)	286 (2.1)>	42 (5.3)	294 (2.1)
	1990	8 (3.4)	276 (5.3)	60 (3.7)	274 (2.1)	32 (4.3)	293 (3.8)
Bottom One-Third	1992	14 (2.7)>	243 (4.3)	49 (4.2)<	243 (2.2)	37 (4.5)>	252 (2.0)
	1990	4 (2.2)	249 (6.9)	77 (5.6)	242 (2.5)	19 (5.0)	250 (5.9)
High Ability	1992	9 (3.4)	293 (9.3)	42 (3.3)	296 (2.3)>	49 (4.8)	303 (2.0)
	1990	5 (2.3)	291 (4.9)	46 (5.8)	280 (4.2)	49 (6.1)	296 (3.0)
Average Ability	1992	11 (2.4)	262 (3.1)<	57 (3.3)<	266 (1.6)>	32 (2.9)>	266 (2.7)
	1990	4 (2.1)	275 (3.2)	75 (4.0)	258 (2.4)	20 (3.5)	265 (5.5)
Low Ability	1992	14 (2.9)	238 (4.7)	60 (4.5)	244 (2.7)	26 (4.5)	248 (3.2)
	1990	13 (4.2)	251 (4.8)	66 (5.8)	241 (4.5)	21 (5.2)	248 (5.4)
Mixed Ability	1992	16 (3.6)>	256 (3.6)	49 (5.6)	262 (3.0)	35 (5.1)	262 (1.8)
	1990	3 (1.7)	256 (6.3)	59 (7.4)	253 (4.5)	38 (7.4)	262 (3.4)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). When the proportion of students is either 0 percent or 100 percent, the standard error is inestimable. However, percentages 99.5 percent and greater were rounded to 100 percent and percentages less than 0.5 percent were rounded to 0 percent. Percentages may not total 100 percent due to rounding error.

— Sample size insufficient to permit a reliable estimate.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 3.3

Students' Reports on the Frequency with Which They Do Mathematics Problems from Textbooks in Mathematics Class, Grades 4, 8, and 12

		ALMOST EVERY DAY		AT LEAST ONCE A WEEK		LESS THAN WEEKLY	
	Assessment Years	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Do mathematics problems from textbooks							
Grade 4							
Nation	1992	66 (1.3)>	220 (0.8)>	17 (0.9)<	221 (1.5)>	17 (1.0)	209 (1.7)
	1990	59 (1.6)	215 (1.2)	26 (1.2)	215 (1.7)	15 (1.0)	204 (2.3)
Top One-Third	1992	67 (2.5)>	237 (0.9)>	20 (1.8)<	240 (2.0)	13 (1.3)	230 (2.2)
	1990	57 (3.5)	230 (1.6)	28 (2.7)	231 (3.1)	15 (2.4)	223 (2.3)
Bottom One-Third	1992	66 (1.8)	200 (1.4)	14 (1.1)<	191 (1.7)	21 (1.4)	189 (1.8)
	1990	59 (2.2)	197 (2.6)	22 (1.2)	195 (2.5)	19 (1.8)	185 (2.7)
Grade 8							
Nation	1992	85 (0.9)>	271 (1.0)	10 (0.7)>	252 (1.8)	5 (0.4)	246 (2.5)
	1990	74 (1.8)	268 (1.2)	20 (1.1)	250 (1.7)	6 (0.9)	242 (5.2)
Top One-Third	1992	91 (1.2)>	291 (1.4)>	7 (1.0)<	272 (3.3)	2 (0.5)	260 (7.8)
	1990	81 (3.7)	284 (1.3)	15 (2.6)	262 (2.8)	4 (1.3)	273(16.1)
Bottom One-Third	1992	79 (1.6)>	249 (1.0)	14 (1.3)<	234 (2.1)	6 (0.7)	231 (2.6)
	1990	69 (3.0)	248 (1.9)	22 (2.1)	236 (3.6)	9 (1.6)	227 (3.8)
Grade 12 – Taking Math†							
Nation	1992	88 (0.9)>	308 (0.9)	8 (0.6)<	295 (3.4)	4 (0.6)	297 (3.7)
	1990	82 (1.4)	305 (1.4)	12 (1.0)	300 (3.1)	6 (0.7)	294 (4.4)
Top One-Third	1992	88 (1.8)	323 (1.4)	7 (1.1)	317 (5.2)	5 (1.1)	307 (6.2)
	1990	84 (3.6)	320 (1.9)	12 (1.9)	316 (4.7)	5 (2.4)	307 (3.9)
Bottom One-Third	1992	86 (1.9)	286 (1.7)	10 (1.3)	271 (3.2)	5 (1.0)	281 (7.0)
	1990	79 (2.4)	282 (2.1)	14 (1.5)	275 (3.9)	6 (1.3)	265 (7.8)

† Sixty-four percent of the twelfth-grade students reported that they were taking a mathematics class at the time of the assessment.

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the proficiencies appear in parentheses. It can be said with 95 percent certainty for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 3.4

Students' Reports on the Frequency with Which They Do Mathematics Problems from Worksheets in Mathematics Class, Grades 4, 8, and 12

	Assessment Years	ALMOST EVERY DAY		AT LEAST ONCE A WEEK		LESS THAN WEEKLY	
		Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Do mathematics problems on worksheets							
Grade 4							
Nation	1992	44 (1.3)>	218 (1.1)>	37 (0.9)<	220 (1.0)>	20 (0.9)	217 (1.2)>
	1990	35 (1.4)	214 (1.4)	44 (1.4)	214 (1.4)	20 (1.2)	210 (1.8)
Top One-Third	1992	46 (2.0)	236 (1.3)>	37 (1.4)	238 (1.0)>	17 (1.4)	235 (1.6)>
	1990	39 (3.5)	229 (1.7)	44 (3.3)	231 (2.1)	17 (2.3)	227 (3.0)
Bottom One-Third	1992	41 (1.8)>	194 (1.5)	36 (1.7)<	199 (1.5)	23 (1.6)	196 (2.0)
	1990	31 (1.5)	190 (2.4)	44 (2.0)	196 (2.9)	24 (1.9)	195 (2.7)
Grade 8							
Nation	1992	22 (1.3)	257 (2.2)>	42 (1.1)	268 (1.3)>	36 (1.5)	274 (1.2)
	1990	18 (1.6)	248 (2.7)	45 (1.7)	261 (3.1)	37 (2.3)	272 (1.6)
Top One-Third	1992	16 (2.4)	283 (4.2)>	41 (2.2)	287 (1.7)>	43 (2.9)	294 (1.4)
	1990	14 (2.2)	268 (2.6)	42 (3.5)	278 (2.1)	44 (4.4)	287 (2.5)
Bottom One-Third	1992	27 (1.9)	238 (1.9)	41 (1.9)<	246 (1.3)	32 (1.8)	251 (1.5)
	1990	22 (2.6)	231 (3.8)	50 (2.6)	246 (2.1)	28 (3.5)	251 (3.0)
Grade 12 – Taking Math†							
Nation	1992	16 (0.9)>	293 (2.6)	37 (1.1)	302 (1.2)	47 (1.5)	315 (1.1)
	1990	12 (1.2)	289 (3.7)	42 (2.2)	299 (1.8)	46 (2.7)	312 (1.7)
Top One-Third	1992	13 (1.4)	313 (4.4)	35 (1.8)	318 (1.8)	52 (2.2)	326 (1.7)
	1990	10 (3.1)	307 (3.5)	41 (4.9)	312 (2.5)	50 (6.2)	327 (2.0)
Bottom One-Third	1992	22 (2.3)>	277 (3.5)>	41 (1.6)	280 (1.7)	36 (3.0)	293 (2.2)
	1990	14 (1.8)	265 (3.2)	47 (3.0)	279 (2.2)	39 (3.5)	287 (3.8)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the proficiencies appear in parentheses. It can be said with 95 percent certainty for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Small-Group Activities and Reports/Projects

Developing students mathematically so they can understand and be able to make sense out of problem situations is central to reform efforts in mathematics education. Frequent suggestions to foster such understanding include using cooperative learning techniques, employing manipulative materials in the learning situation, and engaging students in conducting and reporting problem-oriented projects.[30] Such instructional approaches encourage students to be active participants in the classroom learning process, where they can learn mathematics through various application experiences.

Table 3.5 contains teachers' reports about using small-group activities and projects in their classrooms. The 1992 data indicate that the majority of students participated in small-group activities in mathematics class at least once a week. Small-group activities were more prevalent in the fourth than the eighth grade, with 63 compared to 51 percent of the students, respectively, involved in such activities at least once a week. These figures remained essentially constant between 1990 and 1992, though the percentage of students being assigned no small-group activities rose significantly. Average mathematics proficiency for the nation and for students within ability-grouped classes was not systematically related to the frequency of the use of small-group methods at either grade 4 or grade 8.

Table 3.6 contains students' reports on how often they felt that they had worked in small groups in their mathematics classes. Fourth graders perceived less small-group work than their teachers. In 1992, 37 percent reported such work at least weekly (26 percent fewer than reported by the teachers) and 44 percent reported never or hardly ever being involved in small-group work in mathematics class (an even bigger discrepancy compared to the 9 percent reported by teachers). At both grades 4 and 8, students who reported that they were involved in some small-group work (but less than once a week) had a significantly higher average proficiency level than those reporting more or less use of this teaching method in their class. Significantly more eighth graders in 1992 than in 1990, 36 compared to 28 percent, reported at least weekly involvement in small-group activities.

Forty-two percent of the grade 12 students enrolled in a mathematics class reported some small-group work on a weekly basis. This was a higher amount of small-group involvement than reported by either of the other two grades and a significant increase in the application of this teaching method since 1990.

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[30] Good, T. L., Mulryan, C., & McCaslin, M., "Grouping for Instruction in Mathematics: A Call for Programmatic Research on Small-Group Processes." In D. A. Grouws, *Handbook of Research on Mathematics Teaching and Learning* (New York, NY: Macmillan, 1992).

Davidson, N. (Ed.), *Cooperative Learning in Mathematics* (Menlo Park, CA: Addison-Wesley Publishing Company, 1990).

Table 3.5

Teachers' Reports on the Frequency of Small-Group Activities in Mathematics Class, Grades 4 and 8

		AT LEAST WEEKLY		LESS THAN ONCE A WEEK		NEVER OR HARDLY EVER	
	Assessment Years	**Percentage of Students**	**Average Proficiency**	**Percentage of Students**	**Average Proficiency**	**Percentage of Students**	**Average Proficiency**
About how often do students in this class do the following types of activities for mathematics class?							
Work in small groups							
Grade 4							
Nation	1992	63 (2.5)	218 (1.3)>	28 (2.0)	218 (1.6)	9 (1.3)>	218 (2.4)
	1990	62 (3.1)	213 (1.3)	33 (3.1)	216 (1.8)	5 (1.0)	215 (3.6)
Top One-Third	1992	66 (3.7)	237 (1.3)	25 (3.4)	233 (1.5)	9 (2.5)	235 (2.4)
	1990	54 (6.5)	231 (2.2)	39 (6.1)	227 (2.5)	7 (2.2)	227 (5.4)
Bottom One-Third	1992	63 (4.2)	195 (1.8)	30 (3.5)	199 (1.9)	8 (1.9)	197 (3.5)
	1990	67 (5.0)	193 (2.1)	26 (4.9)	198 (2.3)	6 (2.0)	198 (4.3)
High Ability	1992	71 (6.8)	240 (3.5)	15 (5.5)	230 (4.2)	14 (4.7)>	231 (5.8)
	1990	60(10.3)	242 (5.5)	38(10.6)	226 (6.5)	2 (1.9)	—
Average Ability	1992	58 (4.2)	222 (1.8)>	31 (3.5)	221 (1.9)	11 (2.1)	220 (3.1)
	1990	60 (4.3)	212 (2.4)	31 (4.3)	217 (2.5)	9 (2.6)	219 (5.5)
Low Ability	1992	65 (5.4)	193 (2.7)	27 (4.5)	205 (3.7)	8 (2.9)	192 (8.7)
	1990	71 (6.7)	204 (4.3)	26 (6.8)	196 (5.3)	3 (1.8)	196(13.8)
Mixed Ability	1992	66 (4.0)	217 (1.5)	28 (3.4)	216 (2.5)	7 (1.5)	218 (5.2)
	1990	62 (5.9)	211 (2.5)	34 (5.8)	216 (2.6)	5 (1.7)	211 (4.6)
Grade 8							
Nation	1992	51 (2.5)	270 (1.5)>	32 (2.4)	268 (2.1)	17 (2.2)>	268 (2.7)
	1990	50 (4.1)	261 (2.1)	42 (3.9)	264 (2.3)	9 (2.0)	277 (4.5)
Top One-Third	1992	51 (5.4)	289 (2.7)>	31 (4.6)	287 (1.6)	18 (3.4)	290 (2.2)
	1990	45 (8.5)	279 (2.1)	43 (7.4)	279 (3.2)	12 (3.8)	291 (7.9)
Bottom One-Third	1992	46 (4.6)	245 (2.6)	33 (4.4)	248 (2.0)	22 (4.1)>	249 (2.7)
	1990	53 (6.4)	242 (2.8)	40 (6.2)	244 (4.4)	7 (3.6)	254 (5.4)
High Ability	1992	56 (3.4)	302 (2.4)>	29 (3.7)	294 (2.9)	15 (2.3)	299 (4.0)
	1990	46 (5.1)	284 (3.9)	44 (5.0)	290 (3.2)	10 (3.1)	298 (5.8)
Average Ability	1992	49 (4.4)	265 (2.0)	32 (3.4)	267 (2.5)	20 (3.3)	266 (2.9)
	1990	46 (4.8)	258 (3.1)	44 (5.1)	258 (2.8)	10 (3.2)	282 (6.1)
Low Ability	1992	52 (5.1)	244 (2.3)	32 (4.9)	243 (3.1)	16 (3.5)	245 (4.5)
	1990	49 (7.4)	238 (3.9)	39 (7.0)	250 (5.8)	12 (4.3)	248 (7.3)
Mixed Ability	1992	48 (4.5)	260 (2.6)	36 (5.6)	264 (3.3)	17 (4.1)	258 (4.0)
	1990	55 (8.7)	257 (4.6)	40 (8.9)	255 (4.9)	5 (3.3)	264(12.9)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). When the proportion of students is either 0 percent or 100 percent, the standard error is inestimable. However, percentages 99.5 percent and greater were rounded to 100 percent and percentages less than 0.5 percent were rounded to 0 percent. Percentages may not total 100 percent due to rounding error.

— Sample size insufficient to permit reliable estimate.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 3.6

Students' Reports on the Frequency of Small-Group Activities and Projects in Mathematics Class, Grades 4, 8, and 12

		AT LEAST WEEKLY		LESS THAN ONCE A WEEK		NEVER OR HARDLY EVER	
	Assessment Years	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
In mathematics class, how often do you do each of the following?							
Work in small groups							
Grade 4							
Nation	1992	37 (1.0)	214 (1.0)>	19 (0.7)	229 (1.4)>	44 (1.1)	218 (0.8)>
	1990	32 (1.6)	208 (1.3)	21 (1.0)	224 (1.6)	47 (1.8)	212 (1.3)
Top One-Third	1992	34 (1.8)>	233 (1.3)	25 (1.5)	244 (1.6)>	41 (2.3)	235 (1.1)>
	1990	25 (2.7)	230 (2.6)	28 (2.0)	234 (2.6)	47 (2.9)	226 (2.1)
Bottom One-Third	1992	40 (1.6)	192 (1.4)	14 (0.9)	203 (2.3)	46 (1.5)	199 (1.3)
	1990	38 (1.9)	187 (2.0)	15 (1.2)	204 (3.5)	47 (2.1)	197 (2.2)
Grade 8							
Nation	1992	36 (1.1)>	266 (1.3)	26 (0.9)	272 (1.3)	38 (1.6)	267 (1.2)>
	1990	28 (2.3)	260 (2.6)	27 (1.3)	267 (1.8)	45 (2.8)	262 (1.4)
Top One-Third	1992	34 (2.6)	289 (2.3)>	28 (1.6)	288 (1.4)	38 (3.1)	290 (1.9)>
	1990	27 (5.1)	279 (1.9)	30 (1.8)	282 (2.5)	43 (5.7)	280 (2.0)
Bottom One-Third	1992	36 (1.4)	241 (1.8)	21 (1.5)	251 (1.9)	43 (2.2)	247 (1.3)
	1990	31 (3.6)	239 (2.7)	25 (2.3)	252 (3.1)	44 (4.0)	243 (2.3)
Grade 12 – Taking Math†							
Nation	1992	42 (1.1)>	308 (1.3)	22 (1.0)	308 (1.6)	36 (1.3)<	304 (1.7)
	1990	35 (1.7)	305 (1.9)	23 (1.3)	307 (2.3)	42 (1.8)	302 (2.0)
Top One-Third	1992	42 (2.4)	323 (1.9)	22 (1.7)	321 (1.9)	36 (2.7)	320 (2.6)
	1990	35 (3.2)	318 (2.6)	24 (2.0)	318 (2.6)	41 (3.9)	320 (2.7)
Bottom One-Third	1992	45 (2.2)>	286 (2.2)	20 (1.5)	290 (3.2)	35 (2.1)<	279 (1.9)
	1990	30 (3.5)	281 (3.3)	25 (2.5)	284 (3.2)	44 (2.6)	277 (3.3)

† Sixty-four percent of the twelfth-grade students reported that they were taking a mathematics class at the time of the assessment.

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Tables 3.7 and 3.8 present teachers' and students' reports on the frequency with which students were involved in writing reports or doing mathematics projects. In 1992, grade 4 teachers reported that 82 percent of their students never or hardly ever wrote reports or completed projects in mathematics class. (Fourth graders were not asked to respond to the corresponding question about their report writing or project work.)

For eighth graders, teachers reported assigning reports and projects even less frequently than they did in 1990. In both 1990 and 1992, teachers reported rare use of this instructional approach on a weekly basis. However, less than weekly use was reported for 55 percent of the students in 1990 and this decreased to only 21 percent in 1992. Compared to 43 percent in 1990, teachers in 1992 reported that 78 percent of their eighth-grade students never or hardly ever wrote reports or completed projects in mathematics class. This was consistent with the 77 percent of the students so reporting. At grade 12, 82 percent of the students enrolled in mathematics classes reported never or hardly ever having to write a report or complete a project. The frequency of both small-group work and mathematics projects within each grade was fairly consistent across the top one-third and bottom one-third of the schools, as reported both by teachers and students.

Table 3.7

Teachers' Reports on the Frequency of Projects in Mathematics Class, Grades 4 and 8

		AT LEAST WEEKLY		LESS THAN ONCE A WEEK		NEVER OR HARDLY EVER	
	Assessment Years	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
About how often do students in this class do the following types of activities for mathematics class?							
Write reports or do mathematics projects							
Grade 4							
Nation	1992	1 (0.4)	227 (6.8)	17 (2.0)	219 (3.2)	82 (2.1)	218 (0.9)
Top One-Third	1992	2 (1.1)	240 (5.3)	14 (3.8)	246 (4.8)	83 (4.2)	234 (1.2)
Bottom One-Third	1992	1 (0.4)	197 (6.8)	22 (3.7)	196 (2.6)	78 (3.7)	197 (1.7)
High Ability	1992	5 (3.8)	243 (5.6)	18 (7.0)	252(12.0)	76 (7.4)	234 (2.4)
Average Ability	1992	1 (0.3)	234 (5.6)	16 (2.7)	222 (3.6)	83 (2.7)	221 (1.3)
Low Ability	1992	0 (0.4)	—	15 (4.1)	183 (3.8)	85 (4.0)	197 (2.2)
Mixed Ability	1992	1 (0.6)	204 (6.7)	18 (3.2)	217 (2.8)	80 (3.3)	217 (1.5)
Grade 8							
Nation	1992	1 (0.3)	254(10.6)	21 (1.9)<	267 (2.2)	78 (2.0)>	269 (1.2)
	1990	2 (1.1)	252 (8.1)	55 (4.2)	263 (2.4)	43 (4.4)	265 (2.2)
Top One-Third	1992	1 (0.6)	282(10.0)	19 (3.8)<	289 (2.9)>	80 (3.9)>	289 (1.8)
	1990	2 (1.4)	287 (7.4)	56 (7.7)	279 (2.1)	42 (8.6)	283 (2.5)
Bottom One-Third	1992	1 (0.6)	239 (4.1)	19 (2.7)<	244 (2.5)	79 (2.7)>	248 (1.2)
	1990	5 (2.9)	241(12.7)	70 (6.7)	244 (2.6)	25 (6.0)	242 (5.8)
High Ability	1992	0 (0.1)	—	15 (2.3)<	301 (4.4)>	85 (2.3)>	299 (1.9)
	1990	2 (0.9)	290 (9.0)	59 (6.1)	286 (3.0)	39 (6.1)	290 (3.3)
Average Ability	1992	1 (0.6)	262(17.1)	23 (3.0)<	268 (2.5)>	76 (3.1)>	265 (1.5)
	1990	2 (1.1)	250(20.6)	58 (5.1)	259 (2.5)	40 (5.3)	262 (3.9)
Low Ability	1992	0 (0.2)	—	19 (3.8)<	246 (3.9)	81 (3.8)>	244 (2.6)
	1990	4 (2.3)	232(10.9)	44 (7.3)	241 (4.7)	52 (7.7)	247 (5.1)
Mixed Ability	1992	1 (0.8)	244(12.6)	44 (4.1)<	258 (2.8)	75 (4.1)>	262 (1.8)
	1990	4 (2.5)	251(28.8)	55 (8.2)	254 (4.3)	41 (8.5)	260 (4.0)

>The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

<The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 3.8

Students' Reports on the Frequency of Projects in Mathematics Class, Grades 4, 8, and 12

	Assessment Years	AT LEAST WEEKLY Percentage of Students	AT LEAST WEEKLY Average Proficiency	LESS THAN ONCE A WEEK Percentage of Students	LESS THAN ONCE A WEEK Average Proficiency	NEVER OR HARDLY EVER Percentage of Students	NEVER OR HARDLY EVER Average Proficiency
Write reports or do mathematics projects							
Grade 4							
Nation	1992	5 (0.4)<	242 (3.0)	18 (0.8)	266 (1.7)	77 (0.9)>	270 (1.0)>
	1990	10 (0.8)	242 (3.4)	19 (1.3)	267 (2.4)	70 (1.5)	264 (1.2)
Top One-Third	1992	3 (0.6)<	277 (6.5)	16 (1.4)	287 (2.6)	81 (1.5)	290 (1.4)>
	1990	6 (0.8)	260 (7.4)	19 (2.8)	286 (3.4)	74 (3.1)	281 (1.6)
Bottom One-Third	1992	8 (0.8)<	226 (2.8)	19 (1.2)	244 (1.5)	73 (1.6)>	248 (1.1)
	1990	15 (1.7)	227 (3.9)	22 (2.1)	250 (3.4)	64 (3.1)	246 (1.8)
Grade 12 – Taking Math†							
Nation	1992	3 (0.3)<	277 (4.0)<	15 (0.7)<	301 (2.2)	82 (0.8)>	309 (0.9)
	1990	9 (1.0)	292 (4.2)	20 (1.4)	304 (2.6)	71 (1.5)	305 (1.3)
Top One-Third	1992	2 (0.4)<	282 (8.4)<	14 (1.4)	318 (2.5)	84 (1.5)>	323 (1.6)
	1990	8 (1.6)	314 (5.5)	17 (1.9)	318 (3.8)	75 (2.1)	319 (1.8)
Bottom One-Third	1992	5 (0.7)<	263 (6.5)	18 (1.6)	279 (3.1)	77 (1.7)>	287 (1.5)
	1990	10 (1.4)	268 (3.6)	21 (2.6)	286 (4.3)	69 (2.6)	280 (2.3)

† Sixty-four percent of the twelfth-grade students reported that they were taking a mathematics class at the time of the assessment.

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Manipulatives

Since the mid-1960s, mathematics educators have been promoting the use of manipulative materials in mathematics learning. Such materials include Cuisenaire™ rods, geometric shapes, geoboards, multibase arithmetic blocks, fraction materials, and a host of measuring materials.[31] In an attempt to collect some information about the degree to which such materials were used on a regular basis, teachers and students were asked to report on how often several types of manipulable materials were used in their mathematics classes.

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[31] Biggs, E. E., & MacLean, J. R., *Freedom to Learn: An Active Approach to Mathematics* (Don Mills, Ontario: Addison-Wesley of Canada, 1969).

Dougherty, B. J. & Scott, L., "Curriculum: A Vision for Early Childhood Mathematics." In R. J. Jensen, *Research Ideas for the Classroom: Early Childhood Mathematics* (New York, NY: Macmillan, 1993).

Grade 4 teachers were asked to indicate the frequency with which their students worked with rulers, counting blocks, or geometric shapes. As shown in Table 3.9, they showed stability in their responses between the 1990 and 1992 assessments. In 1992, teachers reported that 44 percent of the fourth graders worked with these types of materials at least weekly, 46 percent worked with them less than once a week, and 10 percent worked with them never or hardly ever. The associated proficiency averages for these three frequency categories were not significantly different. The percentages reported for the top and bottom one-third of the schools as well as for the differing ability-grouped classes were relatively consistent.

Grade 8 teachers were asked to report the frequency with which students worked with measuring instruments or geometric solids. Here teachers reported that 8 percent of the students worked with these materials at least once a week, 50 percent worked with them less than once a week, and 42 percent worked with them never or hardly ever. Again, no significant differences emerged between the top and bottom one-third schools or any of the ability-grouped classes.

Table 3.10 contains students' reports for the same questions. While grade 4 students reported less frequent use of manipulatives than their teachers, the overall patterns in the data were similar. Even though the percentages of students reporting daily use were comparable, there was an indication that fourth graders in top one-third schools had more access to manipulatives than their counterparts in bottom one-third schools. In top one-third schools, 31 percent of the students reported using these materials at least sometimes (less than weekly) and 35 percent reported never or hardly ever using such materials. In bottom one-third schools, 17 percent of the students reported using these materials at least sometimes and 46 percent said that they never or hardly ever did. Eighth-grade students reported higher percentages than their teachers in the "at least weekly" and "never or hardly ever" categories and lower percentages in the "less than once a week" category. When asked about how often they had an opportunity to work with measuring instruments or geometric solids, twelfth graders taking mathematics classes provided reports almost identical to those at grade 8. The percentages for each of the frequency categories were 20 percent weekly, 24 percent less than once a week, and 57 percent never or hardly ever.

Table 3.9

Teachers' Reports on the Frequency with Which Students Do Work with Objects Like Rulers, Counting Blocks, or Geometric Shapes in Mathematics Class at Grade 4, and with Measuring Instruments or Geometric Solids at Grade 8

		AT LEAST WEEKLY		LESS THAN ONCE A WEEK		NEVER OR HARDLY EVER	
	Assessment Years	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Work with rulers, counting blocks, or geometric shapes							
Grade 4							
Nation	1992	44 (2.6)	218 (1.7)	46 (2.6)	218 (1.4)	10 (1.6)>	220 (2.3)
	1990	51 (3.7)	215 (1.6)	47 (3.7)	215 (1.7)	2 (0.7)	202 (8.0)
Top One-Third	1992	47 (4.6)	236 (1.9)	49 (4.7)	235 (1.3)>	4 (1.9)	237 (3.1)
	1990	43 (5.8)	231 (2.4)	56 (5.8)	228 (2.3)	1 (0.5)	230 (3.5)
Bottom One-Third	1992	46 (4.1)	196 (2.4)	47 (4.2)	196 (1.7)	7 (1.6)	201 (4.4)
	1990	51 (7.9)	196 (2.2)	46 (7.8)	196 (2.7)	4 (1.5)	182 (6.9)
High Ability	1992	38 (7.7)	246 (5.9)	41 (8.6)	233 (2.8)	21 (6.6)	233 (6.3)
	1990	50 (9.2)	237 (5.7)	50 (9.2)	235 (7.3)	0 (0.0)	—
Average Ability	1992	40 (4.2)	223 (2.4)	47 (3.8)	221 (1.7)	12 (2.4)>	220 (3.2)
	1990	53 (4.9)	215 (2.6)	42 (4.5)	215 (2.7)	4 (1.8)	208(11.3)
Low Ability	1992	45 (4.8)	193 (3.3)	50 (4.8)	199 (3.2)	5 (1.6)	193(11.1)
	1990	59 (8.0)	203 (4.0)	37 (7.8)	205 (7.4)	4 (1.8)	188(10.8)
Mixed Ability	1992	48 (3.3)	217 (1.6)	45 (3.8)	217 (2.1)	7 (2.3)>	217 (2.9)
	1990	44 (5.8)	211 (2.4)	56 (5.8)	214 (2.5)	0 (0.4)	—
Work with measuring instruments or geometric solids							
Grade 8							
Nation	1992	8 (1.1)	272 (3.2)	50 (3.0)	266 (1.4)	42 (3.0)	272 (2.0)
Top One-Third	1992	8 (2.2)	298 (4.6)	41 (5.4)	284 (2.5)	51 (5.8)	291 (2.1)
Bottom One-Third	1992	7 (1.8)	250 (4.3)	53 (4.6)	247 (2.0)	40 (4.7)	246 (2.4)
High Ability	1992	9 (1.9)	306 (5.7)	36 (4.5)	299 (2.7)	56 (4.9)	300 (2.1)
Average Ability	1992	9 (2.2)	268 (3.6)	56 (4.1)	264 (1.9)	34 (3.6)	267 (2.4)
Low Ability	1992	6 (1.8)	236 (8.9)	52 (6.2)	245 (2.4)	42 (6.1)	244 (2.6)
Mixed Ability	1992	7 (2.5)	259 (3.8)	50 (5.0)	261 (1.9)	43 (5.5)	262 (2.7)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). When the proportion of students is either 0 percent or 100 percent, the standard error is inestimable. However, percentages 99.5 percent and greater were rounded to 100 percent and percentages 0.5 percent were rounded to 0 percent. Percentages may not total 100 percent due to rounding error.

— Sample size insufficient to permit a reliable estimate. The question about working with instruments and solids was not asked in 1990.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 3.10

Students' Reports on the Frequency with Which They Do Work with Objects Like Rulers, Counting Blocks, or Geometric Shapes in Mathematics Class at Grade 4, and with Measuring Instruments or Geometric Solids at Grades 8 and 12

		AT LEAST WEEKLY		LESS THAN ONCE A WEEK		NEVER OR HARDLY EVER	
	Assessment Years	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Work with rulers, counting blocks, or geometric shapes							
Grade 4							
Nation	1992	34 (1.2)<	216 (1.3)>	24 (0.8)	227 (0.9)	41 (1.2)>	216 (1.0)>
	1990	41 (1.5)	210 (1.3)	25 (1.1)	225 (1.7)	34 (1.3)	208 (1.3)
Top One-Third	1992	34 (2.2)	235 (1.3)>	31 (1.5)	240 (1.5)	35 (1.9)	234 (1.0)>
	1990	42 (2.9)	228 (2.2)	31 (1.9)	236 (2.0)	27 (2.7)	224 (2.4)
Bottom One-Third	1992	37 (2.0)	194 (1.6)	17 (1.0)	203 (2.0)	46 (2.3)	196 (1.4)
	1990	42 (2.1)	189 (1.7)	19 (1.6)	211 (3.3)	39 (2.5)	192 (1.8)
High Ability	1992	33 (3.9)	234 (3.5)	31 (2.8)	246 (4.4)	36 (3.4)	234 (2.7)
	1990	46 (4.2)	238 (5.8)	28 (3.3)	244 (5.2)	26 (4.6)	224 (5.7)
Average Ability	1992	32 (1.7)<	221 (2.3)>	25 (1.8)	227 (1.7)	42 (2.1)>	219 (1.3)
	1990	41 (2.4)	212 (2.1)	27 (2.1)	223 (2.7)	32 (2.0)	212 (3.0)
Low Ability	1992	36 (2.7)	191 (3.1)	18 (1.8)	203 (3.0)	46 (2.9)	197 (2.9)
	1990	40 (4.3)	198 (4.5)	21 (3.0)	216 (6.8)	39 (4.7)	200 (4.0)
Mixed Ability	1992	38 (2.0)	214 (1.8)>	25 (1.4)	224 (1.5)	38 (2.0)	215 (1.9)>
	1990	37 (2.8)	207 (2.1)	27 (2.1)	227 (3.3)	36 (3.6)	208 (1.9)
Work with measuring instruments or geometric solids							
Grade 8							
Nation	1992	20 (1.1)	265 (1.6)	28 (0.9)	273 (1.3)	52 (1.4)	266 (1.0)
Top One-Third	1992	19 (1.8)	290 (1.9)	32 (1.1)	290 (1.9)	50 (2.1)	288 (1.5)
Bottom One-Third	1992	19 (1.4)	240 (1.8)	23 (1.6)	251 (1.8)	58 (2.4)	245 (1.2)
High Ability	1992	16 (1.6)	295 (3.7)	28 (1.6)	300 (2.6)	56 (2.2)	300 (2.0)
Average Ability	1992	22 (1.6)	265 (2.2)	29 (1.3)	271 (2.0)	49 (2.1)	263 (1.4)
Low Ability	1992	20 (1.9)	241 (3.3)	26 (2.3)	251 (3.6)	54 (2.8)	243 (2.0)
Mixed Ability	1992	23 (2.5)	261 (2.2)	27 (2.4)	269 (1.9)	50 (2.9)	258 (1.7)
Grade 12 – Taking Math†							
Nation	1992	20 (1.0)	301 (1.7)	24 (0.8)	310 (1.2)	57 (1.2)	307 (1.1)
Top One-Third	1992	16 (1.5)	320 (2.7)	25 (1.4)	323 (1.9)	59 (1.8)	321 (1.8)
Bottom One-Third	1992	24 (1.8)	282 (2.4)	20 (1.1)	290 (2.4)	56 (2.0)	283 (1.8)

† Sixty-four percent of the twelfth-grade students reported that they were taking a mathematics class at the time of the assessment.

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Assessing and Testing in Mathematics Classes

In NAEP's 1990 assessment, the frequency of testing taking place in the classroom was related to achievement, with more frequent testing generally associated with lower levels of student proficiency.[32] As a result of that finding and mathematics educators' increased emphasis and interest in assessment,[33] additional questions about assessment strategies were asked in 1992. The data from teachers' and students' responses are presented in Tables 3.11 and 3.12.

When asked how often they used multiple-choice exams to assess student progress in their mathematics classes, teachers of 51 percent of the grade 4 students and 66 percent of the grade 8 students responded only yearly or never. When the assessment approach was problem sets, the teachers' response shifted to once or twice a week for 53 percent of the grade 4 students and 58 percent of the grade 8 students. Problem sets are a set of one to five problem situations, word problems from the textbook, or written or verbal questions constructed by the teacher.

When asked about the use of short (ranging from a phrase to a sentence) or long written responses to problems (several sentences or a paragraph), teachers responded in a bimodal pattern. At grade 4, teachers of 44 percent of the students responded at least monthly, while teachers of 40 percent of the students responded yearly or never. At grade 8, a similar pattern occurred with 44 percent of the students asked to provide written responses at least monthly and 33 percent of the students never or hardly ever required to do so.

The fourth question on assessment probed teachers' use of projects, portfolios, or presentations as a form of assessing student progress. Grade 4 teachers' responses indicated that more than half, 54 percent, of the students never or hardly ever participated in these assessment activities, and at grade 8, 47 percent of the students never or hardly ever did so. These findings are worth highlighting given the heavy emphasis on moving to alternative forms of assessment as a way of evaluating student progress.[34] Interestingly, no systematic relationship was seen between average proficiency and the frequency with which students were given the opportunity to participate in alternative-type assessments.

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[32] Mullis, I. V.S., Dossey, J. A., Owen, E. H., & Phillips, G. W., *The State of Mathematics Achievement: NAEP's 1990 Assessment of the Nation and the Trial State Assessment of the States* (Washington, DC: National Center for Education Statistics, 1991).

[33] *Measuring What Counts* (Washington, DC: Mathematical Sciences Education Board, National Research Council, National Academy Press, 1993).

Barton, P. E. and Coley, R. J., *Testing in America's Schools* (Princeton, NJ: Educational Testing Service, 1994).

Students' responses to a question on the frequency of testing in their mathematics classes are contained in Table 3.12. They reported less frequent testing in mathematics classes in 1992 than in 1990. The pattern of average student proficiency by frequency of testing showed that less frequent testing was associated with higher average mathematics proficiency. At both grades 4 and 8, students reported more testing in bottom one-third schools than in top one-third schools.

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[34] Webb, N. L., "Assessment of Students' Knowledge of Mathematics: Steps Toward a Theory." In D. A. Grouws, *Handbook of Research on Mathematics Teaching and Learning* (New York, NY: Macmillan, 1992).

Measuring Up: Prototypes for Mathematics Assessment (Washington, DC: Mathematical Sciences Education Board, National Research Council, National Academy Press, 1993).

Dossey, J. A., Mullis, I. V.S., & Jones, C. O., *Can Students Do Mathematical Problem Solving? Results from Constructed-Response Questions in NAEP's 1992 Mathematics Assessment* (Washington, DC: National Center for Education Statistics, 1993).

Table 3.11

Teachers' Reports on the Frequency of Testing in Mathematics Classes, Grades 4 and 8

Assessment Year – 1992	ONCE OR TWICE A WEEK		ONCE OR TWICE A MONTH		YEARLY OR NEVER	
	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
How often do you use each of the following to assess student progress in mathematics class?						
Multiple-Choice Tests						
Grade 4						
Nation	6 (1.0)	206 (3.6)	43 (2.8)	220 (1.4)	51 (2.7)	219 (1.3)
Top One-Third	4 (1.5)	233 (6.4)	44 (5.8)	237 (1.8)	52 (5.6)	235 (1.6)
Bottom One-Third	13 (2.9)	195 (3.0)	44 (4.0)	198 (2.0)	43 (4.2)	196 (1.8)
Grade 8						
Nation	4 (1.0)	266 (7.6)	30 (2.5)	263 (2.0)	66 (2.8)	271 (1.3)
Top One-Third	6 (2.1)	294 (7.4)	26 (4.4)	285 (3.6)	68 (5.3)	290 (1.6)
Bottom One-Third	6 (1.8)	235 (4.5)	38 (4.0)	246 (2.2)	57 (4.6)	248 (1.5)
Problem Sets						
Grade 4						
Nation	53 (2.8)	218 (1.1)	39 (2.3)	220 (1.3)	9 (1.4)	212 (2.7)
Top One-Third	52 (5.2)	234 (1.5)	42 (5.0)	238 (1.9)	6 (1.9)	233 (3.0)
Bottom One-Third	57 (4.0)	197 (1.8)	33 (3.3)	197 (2.5)	10 (2.2)	194 (2.6)
Grade 8						
Nation	58 (2.3)	272 (1.5)	32 (2.4)	265 (1.5)	10 (1.7)	263 (3.1)
Top One-Third	64 (4.2)	290 (1.6)	30 (3.7)	287 (1.8)	6 (2.2)	280(13.0)
Bottom One-Third	48 (4.0)	250 (1.6)	37 (3.1)	242 (2.4)	15 (3.3)	249 (4.1)
Short (Phrase/Sentence) or Long (Several Sentences/Paragraph) Written Responses						
Grade 4						
Nation	44 (2.6)	218 (1.7)	16 (1.5)	218 (2.4)	40 (2.0)	219 (1.3)
Top One-Third	44 (4.8)	238 (1.8)	17 (3.3)	234 (1.9)	39 (3.9)	234 (1.1)
Bottom One-Third	48 (5.5)	194 (1.7)	18 (3.4)	197 (2.3)	34 (4.1)	201 (2.8)
Grade 8						
Nation	44 (2.7)	270 (1.5)	22 (2.0)	268 (1.9)	33 (2.7)	268 (2.0)
Top One-Third	44 (4.9)	290 (2.1)	22 (3.8)	287 (2.4)	34 (5.5)	289 (2.4)
Bottom One-Third	44 (3.6)	247 (2.0)	21 (2.8)	244 (3.0)	36 (4.0)	248 (2.1)
Projects, Portfolios, or Presentations						
Grade 4						
Nation	20 (1.7)	218 (2.2)	25 (1.8)	220 (2.3)	54 (2.4)	217 (1.2)
Top One-Third	23 (3.9)	237 (1.9)	25 (4.5)	238 (3.1)	52 (4.5)	234 (1.4)
Bottom One-Third	21 (2.9)	193 (2.3)	30 (3.3)	201 (2.0)	48 (4.2)	196 (2.3)
Grade 8						
Nation	21 (2.0)	268 (2.3)	32 (2.5)	269 (1.5)	47 (2.6)	269 (1.7)
Top One-Third	19 (3.4)	289 (3.9)	33 (5.0)	288 (2.3)	48 (4.7)	290 (2.0)
Bottom One-Third	18 (2.6)	240 (2.3)	31 (3.4)	246 (2.4)	51 (3.8)	249 (1.3)

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 3.12

Students' Reports on How Often They Take Mathematics Tests, Grades 4, 8, and 12

	Assessment Years	ALMOST EVERY DAY		AT LEAST ONCE A WEEK		LESS THAN WEEKLY	
		Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Grade 4							
Nation	1992	9 (0.6)	199 (2.1)>	30 (1.2)<	213 (1.0)	61 (1.5)>	224 (1.0)>
	1990	9 (0.7)	190 (2.5)	43 (1.0)	210 (1.0)	47 (1.2)	220 (1.2)
Top One-Third	1992	5 (0.8)	226 (3.6)	22 (1.9)<	231 (1.2)	73 (2.1)>	239 (1.1)>
	1990	4 (0.9)	210 (6.4)	39 (1.9)	228 (2.0)	57 (1.9)	232 (1.8)
Bottom One-Third	1992	15 (1.4)	184 (2.1)	38 (1.8)<	195 (1.5)	47 (2.3)>	202 (1.5)
	1990	15 (1.8)	176 (3.0)	49 (1.8)	194 (2.0)	36 (1.9)	202 (2.0)
Grade 8							
Nation	1992	6 (0.3)>	248 (2.0)	55 (1.2)<	265 (1.0)	39 (1.3)>	275 (1.3)>
	1990	4 (0.5)	242 (2.9)	65 (2.1)	262 (1.6)	31 (2.1)	268 (1.7)
Top One-Third	1992	3 (0.5)	281 (6.6)	48 (2.2)<	287 (2.1)>	50 (2.5)>	291 (1.0)>
	1990	3 (0.5)	270 (4.4)	64 (5.0)	280 (1.8)	33 (4.7)	283 (1.8)
Bottom One-Third	1992	10 (0.8)>	232 (1.9)	62 (1.8)	246 (1.1)	28 (2.0)	251 (1.7)
	1990	7 (1.0)	228 (4.2)	68 (2.6)	244 (2.3)	26 (2.5)	248 (2.7)
Grade 12 – Taking Math†							
Nation	1992	4 (0.4)	284 (3.7)	57 (1.4)<	302 (1.1)	39 (1.5)>	316 (1.3)
	1990	3 (0.5)	282 (5.6)	68 (2.3)	302 (1.4)	29 (2.3)	311 (2.3)
Top One-Third	1992	3 (0.5)	312 (7.5)	50 (2.1)<	317 (1.4)	47 (2.1)>	327 (2.0)
	1990	2 (0.8)	301(10.7)	67 (4.5)	317 (2.1)	31 (4.1)	324 (2.1)
Bottom One-Third	1992	6 (1.0)	271 (4.7)	66 (2.6)	282 (1.6)	27 (3.1)	292 (3.5)
	1990	6 (1.4)	266 (6.2)	72 (3.0)	280 (1.8)	22 (3.9)	283 (4.3)

† Sixty-four percent of the twelfth-grade students reported that they were taking a mathematics class at the time of the assessment.

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Helping Students Think About Mathematics

Teachers were asked about the frequency with which students engaged in *four activities designed to strengthen* problem-solving skills: writing about how they had solved problems, developing problems for their peers to solve, discussing their problem-solving strategies with classmates, and working on mathematics problems reflecting real-life situations. All of these activities involve the student directly in producing and communicating mathematics ideas,[35] and support the NCTM process standards of problem solving, reasoning, communication, and connecting mathematics to the students' context.

The teachers' responses about requiring students to provide written descriptions of how they solved problems are presented in Table 3.13. Approximately 40 percent of the students in both grades 4 and 8 were never or hardly ever required to write a few sentences about their problem-solving strategies. At the other end of the frequency spectrum, about 20 percent were required to do so weekly.

The second question inquired about how often students developed problems for their classmates to consider. Such an activity, when well done, requires a good deal of student understanding about the necessary information required to solve a problem. It also helps students relate the roles of the types of information provided to the available problem-solving strategies. The responses from grade 4 teachers indicate that 21 percent of the students had a chance to do this weekly, and 31 percent of the students never or hardly ever had such opportunities. The responses of grade 8 teachers indicated that nearly 60 percent of the students never or hardly ever participated in this activity.

Students' average proficiency did not vary significantly in relation to teachers' reports about the frequency of using either of these two activities — students writing about how they solved problems or developing problems for their classmates to solve. Also, teachers reported using these activities with equal frequency in the top and bottom one-third of the schools.

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[35] Schoenfeld, A. H., "Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics." In D. A. Grouws, *Handbook of Research on Mathematics Teaching and Learning* (New York, NY: Macmillan, 1992).

Measuring Up: Prototypes for Mathematics Assessment (Washington, DC: Mathematical Sciences Education Board, National Research Council, National Academy Press, 1993).

Romberg, T. A. & Carpenter, T. P., "Research on Teaching and Learning Mathematics: Two Disciplines of Scientific Inquiry." In M. C. Wittrock, *Handbook of Research on Teaching* (Third Edition), (New York, NY: Macmillan, 1986).

Students in grades 8 and 12 were asked the same two questions about how often they described their problem-solving strategies or developed problems for their classmates to consider. The results are shown in Table 3.14. In the case of writing about how they solved a problem, just like the teachers, 21 percent of the grade 8 students said they did this weekly. However, 62 percent of the students reported that they did so never or hardly ever, while teachers reported 41 percent. Twelfth graders currently taking mathematics reported participation similar to the eighth graders — 68 percent reported never or hardly ever being asked to write about how they solved a problem. At both grades 8 and 12, students who reported writing about their problem solving at least weekly had significantly lower average proficiency than those students who reported writing about their problem solving less frequently.

Approximately 80 percent of both grade 8 and grade 12 students reported never or hardly ever making up problems for their classmates to solve. Those reporting that they did participate in this activity at least weekly had lower performance than their classmates who reported making up problems less frequently. Higher percentages of eighth graders in bottom one-third schools than in top one-third schools reported making up problems for their classmates to solve on at least a weekly basis (13 compared to 5 percent).

It is interesting that in contrast to the teachers' responses, lower-performing students reported more frequent use of these kinds of written activities than their higher-performing counterparts. It may be that teachers perceive these kinds of activities primarily as a useful supplement for students who are having difficulty with their textbooks. Or, maybe these more challenging activities are simply more memorable for the poorer students.

Table 3.13

Teachers' Reports on How Often They Ask Students to Write a Few Sentences About How to Solve a Mathematics Problem and to Make Up Mathematics Problems for Other Students to Solve, Grades 4 and 8

Assessment Year – 1992	AT LEAST WEEKLY		LESS THAN ONCE A WEEK		NEVER OR HARDLY EVER	
	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
About how often do students in this class do the following types of activities for mathematics class?						
Write a few sentences about how they solved a mathematics problem						
Grade 4						
Nation	19 (2.0)	219 (2.3)	36 (1.9)	218 (1.7)	45 (2.3)	218 (1.2)
Top One-Third	23 (4.1)	238 (2.3)	33 (3.3)	237 (1.9)	44 (4.8)	234 (1.2)
Bottom One-Third	23 (3.3)	194 (2.7)	39 (4.9)	198 (2.4)	39 (4.5)	197 (1.6)
Grade 8						
Nation	21 (2.1)	273 (2.3)	38 (2.5)	267 (1.6)	41 (2.5)	268 (1.7)
Top One-Third	23 (3.9)	294 (3.2)	34 (4.4)	284 (2.5)	42 (5.2)	290 (1.9)
Bottom One-Third	17 (2.6)	244 (3.2)	40 (4.6)	248 (1.9)	43 (3.7)	247 (1.7)
Make up mathematics problems for other students to solve						
Grade 4						
Nation	21 (2.1)	217 (1.8)	47 (2.2)	221 (1.4)	31 (2.2)	216 (1.6)
Top One-Third	22 (3.4)	234 (1.9)	51 (3.9)	238 (1.4)	27 (3.5)	233 (1.5)
Bottom One-Third	26 (2.9)	196 (2.0)	41 (3.3)	198 (2.3)	34 (3.5)	196 (2.6)
Grade 8						
Nation	8 (1.2)	268 (3.6)	34 (2.3)	269 (2.1)	59 (2.4)	269 (1.4)
Top One-Third	7 (2.0)	290 (6.3)	33 (4.5)	289 (3.3)	60 (4.7)	289 (1.5)
Bottom One-Third	9 (1.9)	246 (3.7)	27 (2.9)	242 (2.5)	64 (3.8)	249 (1.3)

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 3.14

Students' Reports on How Often They Write Sentences About How to Solve a Mathematics Problem and Make Up Mathematics Problems for Other Students to Solve, Grades 8 and 12

	AT LEAST WEEKLY		LESS THAN ONCE A WEEK		NEVER OR HARDLY EVER	
Assessment Year – 1992	**Percentage of Students**	**Average Proficiency**	**Percentage of Students**	**Average Proficiency**	**Percentage of Students**	**Average Proficiency**
In mathematics class, how often do you do the following?						
Write a few sentences about how they solved a mathematics problem						
Grade 8						
Nation	21 (0.8)	258 (1.3)	18 (0.7)	270 (1.4)	62 (1.1)	271 (1.1)
Top One-Third	18 (1.5)	284 (2.2)	19 (1.2)	289 (2.3)	62 (1.9)	291 (1.5)
Bottom One-Third	26 (1.1)	236 (1.7)	15 (0.8)	249 (2.2)	60 (1.3)	249 (1.0)
Grade 12 — Taking Math†						
Nation	15 (0.8)	298 (2.3)	17 (0.9)	310 (1.6)	68 (1.0)	308 (1.0)
Top One-Third	13 (1.2)	318 (3.2)	19 (1.2)	323 (2.2)	67 (1.7)	322 (1.6)
Bottom One-Third	18 (1.8)	276 (4.1)	16 (1.9)	290 (2.9)	66 (2.4)	285 (1.4)
Make up mathematics problems for other students to solve						
Grade 8						
Nation	8 (0.4)	243 (1.9)	15 (0.7)	268 (1.4)	77 (0.8)	271 (1.0)
Top One-Third	5 (0.5)	271 (3.8)	16 (1.4)	288 (2.5)	79 (1.5)	290 (1.3)
Bottom One-Third	13 (0.9)	229 (2.0)	15 (1.0)	247 (2.2)	72 (1.1)	249 (1.1)
Grade 12 — Taking Math†						
Nation	5 (0.4)	283 (3.7)	9 (0.5)	304 (2.2)	86 (0.8)	308 (1.0)
Top One-Third	3 (0.6)	302 (5.5)	9 (0.8)	319 (4.1)	88 (1.2)	322 (1.5)
Bottom One-Third	8 (0.9)	269 (4.9)	10 (0.8)	285 (3.0)	82 (1.4)	285 (1.5)

† Sixty-four percent of the twelfth-grade students reported that they were taking a mathematics class at the time of the assessment.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error. These questions were not asked at grade 4.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 3.15 presents feedback from fourth and eighth graders' teachers concerning the frequency with which they asked students to discuss solutions to mathematics problems. The teachers' responses reflect a higher level of participation than that seen for the previous two activities, especially at grade 8. At grade 4, teachers reported that 33 percent of the students were involved in this activity daily, 39 percent at least once a week, and 28 percent less than weekly. At grade 8, the pattern was somewhat similar, with 43 percent of the students involved in discussing solutions daily, 32 percent at least once a week, and 25 percent less than weekly. At grade 4, participation in such discussions did not seem to have any noticeable relationship to the students' proficiency scores. At grade 8, students participating in such discussions daily had significantly higher mathematics proficiency scores than students participating less frequently in discussions about problem solving.

Teachers' reports about how often they required students to work and discuss mathematical problems that reflected real-life situations also are shown in Table 3.15. This question was an attempt to derive a measure of the frequency with which students were asked to make connections between the world of their mathematics classroom and the context in which the rest of their life takes place. At both grades 4 and 8, the teachers' responses suggest that the majority of the students had some opportunities to make and discuss such connections in the classrooms. At grade 4, teachers reported that 26 percent of the students participated in such activities daily, with another 48 percent being involved at least once a week. At grade 8, 19 percent of the students participated in such activities daily, with another 51 percent being involved at least once a week. Neither grade showed a relationship between the activity and average proficiency. In the top- and bottom-performing schools, teachers reported roughly the same degree of emphasis on student discussion of mathematics problems and on having them work problems in real-life contexts. (Students were not asked about the frequency of their participation in these two types of discussion activities.)

Table 3.15

Teachers' Reports on How Often They Ask Students to Discuss Solutions to Mathematics Problems with Other Students, and Work and Discuss Mathematics Problems that Reflect Real-Life Situations, Grades 4 and 8

	ALMOST EVERY DAY		AT LEAST ONCE A WEEK		LESS THAN WEEKLY	
Assessment Year – 1992	**Percentage of Students**	**Average Proficiency**	**Percentage of Students**	**Average Proficiency**	**Percentage of Students**	**Average Proficiency**
About how often do students in this class do the following types of activities for mathematics class?						
Discuss solutions to mathematics problems with other students						
Grade 4						
Nation	33 (2.8)	218 (1.6)	39 (1.9)	219 (1.5)	28 (2.1)	218 (1.6)
Top One-Third	27 (3.4)	239 (1.7)	48 (3.5)	235 (1.9)	25 (2.8)	234 (1.4)
Bottom One-Third	33 (4.6)	196 (2.0)	43 (4.4)	198 (2.5)	24 (2.7)	195 (2.6)
Grade 8						
Nation	43 (2.3)	274 (1.7)	32 (2.1)	266 (2.1)	25 (2.1)	262 (1.8)
Top One-Third	46 (3.5)	293 (1.9)	34 (4.0)	289 (2.6)	20 (3.8)	280 (3.1)
Bottom One-Third	40 (4.3)	251 (2.0)	29 (3.9)	240 (2.6)	30 (3.2)	247 (2.7)
Work and discuss mathematics problems that reflect real-life situations						
Grade 4						
Nation	26 (2.1)	217 (1.7)	48 (2.4)	219 (1.4)	27 (2.1)	217 (1.5)
Top One-Third	24 (3.1)	237 (1.7)	53 (5.1)	237 (1.4)	23 (4.5)	232 (1.6)
Bottom One-Third	31 (3.9)	197 (1.9)	44 (3.1)	195 (2.2)	26 (2.6)	198 (2.0)
Grade 8						
Nation	19 (1.6)	269 (2.8)	51 (2.2)	269 (1.3)	29 (2.0)	268 (1.8)
Top One-Third	22 (4.6)	292 (3.8)	55 (4.9)	286 (1.9)	23 (4.3)	292 (2.0)
Bottom One-Third	18 (2.8)	247 (2.6)	47 (4.1)	246 (2.1)	34 (3.7)	249 (2.4)

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Summary

The primary source of mathematics problems for both fourth and eighth graders was the textbook. Teachers reported that 76 percent of the fourth graders and 83 percent of the eighth graders solved problems from their textbooks on a daily basis. This represented a significant increase from 1990 at both grades. At grade 8, students who used their textbooks daily had higher average proficiency than their classmates who did problems from their textbooks less often. In general, students' reports agreed with those of their teachers. At all three grades, students reported an increase between 1990 and 1992 in daily problem solving from textbooks. The percentages of students reporting they did problems from their textbooks almost every day were 66 percent at grade 4, 85 percent at grade 8, and 88 percent at grade 12 for those students in mathematics classes.

There was less agreement between teachers and students about the frequency of small-group activities. Teachers reported this as a frequent activity more for fourth than eighth graders, with 63 percent of the fourth graders compared to 51 percent of the eighth graders engaging in small-group work at least weekly. Fewer students, 37 percent at grade 4, 36 percent at grade 8, and 42 percent at grade 12, reported participating in small-group activities on at least a weekly basis. Both eighth and twelfth graders, however, reported more frequent small-group work in 1992 than in 1990.

Teachers reported that 44 percent of the fourth graders worked at least weekly with rulers, counting blocks, or geometric shapes. This did not vary across the top and bottom one-third performing schools. Fourth graders' reports were similar (34 percent at least weekly with no differences across the top and bottom one-third performing schools).

According to their teachers, students were most frequently assessed with problem sets (53 to 58 percent of the fourth and eighth graders were assessed this way once or twice a week). The majority of the students were hardly ever or never assessed with multiple-choice tests (51 percent of the fourth graders and 66 percent of the eighth graders). For substantial percentages of the students — 40 percent at grade 4 and 33 percent at grade 8 — short or long written responses were hardly ever or never used. About half the students (54 percent at grade 4 and 47 percent at grade 8) were never or hardly ever assessed with projects, portfolios, or presentations.

At all three grades, students who reported being tested more frequently had lower average proficiencies than those who reported less testing.

Students at all three grades reported less frequent testing in 1992 than in 1990. In 1992, at grade 4, 9 percent reported being tested almost daily, 30 percent at least weekly, and 61 percent less than weekly. Weekly testing, however, increased at grade 8 — 6 percent daily, 55 percent weekly, and 39 percent less than weekly. The reports of twelfth graders taking mathematics classes were nearly identical to those of eighth graders (4 percent daily, 57 percent weekly, and 39 percent less than weekly).

In response to a series of questions about how often they used various instructional approaches designed to help students with their problem-solving skills, teachers reported rather infrequent use. Teachers used these activities more often with lower-performing students, perhaps to supplement their textbooks. According to teachers at grades 4 and 8, 45 and 41 percent of the students, respectively, never or hardly ever were asked to write a few sentences about how they solved a mathematics problem. Teachers reported that 31 percent of the grade 4 students and 59 percent of the grade 8 students were never or hardly ever asked to make up mathematics problems for other students to solve. Twenty-eight percent of the fourth graders and 25 percent of the eighth graders were never or hardly ever asked to discuss solutions to mathematics problems with their classmates. Teachers also reported that 27 percent of grade 4 students and 29 percent of their grade 8 counterparts were never or hardly ever asked to work on or discuss mathematics problems that reflected real-life situations. How are students expected to achieve the goals for a process-oriented curriculum centered on the value of studying mathematics and connecting that mathematics to students' daily lives if such activities do not become a regular part of their opportunities to learn?

Students confirmed the infrequent use of these types of activities. Sixty-two percent of the grade 8 students and 68 percent of the grade 12 mathematics students reported that they never or hardly ever were asked to write a few sentences about how they solved a mathematics problem. When asked to report on the frequency with which they had constructed a mathematics problem for their peers to solve, 77 percent of the grade 8 students and 86 percent of the grade 12 mathematics students indicated that they never or hardly ever were asked to do such activities. Students' limited exposure to writing about their problem-solving strategies is of particular concern given the current emphasis and importance placed on communicating ideas in mathematics. Reforms in school mathematics feature such activities as central to instruction and assessment.

4

Calculators and Computers in Mathematics Classes

Another group of NAEP background questions addressed the extent to which calculators and computers were used in school mathematics programs. Parents and school administrators have been cautious about, or even opposed to, implementing wider use of calculators in school classrooms. Some have expressed concern about damaging students' mastery of basic skills in mathematics.[36] In contrast, research shows that the proper use of calculators can enhance learning at all stages.[37] Calculators can take the drudgery out of mathematics and free the learner to concentrate on high-order problem-solving skills. For example, the NCTM standards make a clear statement supporting the important role calculators can play in helping students of all ages to explore, verify, and create mathematics.[38] The NCTM standards call for all students to have access to appropriate calculators throughout their school experiences — in the lower grades a four-function calculator, a scientific calculator in the middle grades, and a graphing calculator thereafter. NAEP provided students with four-function calculators in grade 4 and scientific calculators in grades 8 and 12 for use in completing portions of the mathematics assessments. Consistent with the NCTM recommendations on the role of calculators in the curriculum, the assessment collected information on whether students knew how and when to use the calculator, as well as on the frequency of calculator usage in the school mathematics program. Teachers and students also provided information about the role played by computers in school mathematics programs.

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[36] Campbell, P. F. & Stewart, E. L., "Calculators and Computers." In R. F. Jensen, *Research Ideas for the Classroom: Middle Grades Mathematics* (New York, NY: Macmillan, 1993).

Jensen, R. J. & Williams, B. S., "Technology: Implications for Middle Grades Mathematics." In D. T. Owens, *Research Ideas for the Classroom: Early Childhood Mathematics* (New York, NY: Macmillan, 1993).

Policies for Using Calculators in Mathematics Class

Teachers were asked whether students had unrestricted or restricted use of calculators in their mathematics classes. Their responses in 1992, presented in Table 4.1, show that at grade 4, 95 percent of the nation's students had some form of restrictions placed on their calculator use in mathematics class. By grade 8, the picture changed somewhat, as 70 percent of the students had restricted use of calculators in mathematics class. The 30 percent of eighth graders permitted unrestricted calculator use had significantly higher average proficiency than the 70 percent with restricted use. Eighth graders in top one-third schools had more opportunities to use a calculator in mathematics class than did students in bottom one-third schools — 34 compared to 19 percent were permitted unrestricted use.

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[37] Lacampagne, C.B., *State of the Art, Transforming Ideas for Teaching and Learning Mathematics* (Washington, DC: U.S. Department of Education, Office of Educational Research and Improvement, Office of Research, U.S. Government Printing Office, 1993).

[38] National Council of Teachers of Mathematics, *Professional Standards for Teaching Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 1991).

Kaput, J. J., "Technology and Mathematics Education." In D. A. Grouws, *Handbook of Research on Mathematics Teaching and Learning* (New York, NY: Macmillan, 1992).

Eighth graders in high-ability classrooms were significantly more likely than students in average-, low-, or mixed-ability classrooms to be allowed unrestricted use of calculators.

While little change in calculator-use policies was noted between 1990 and 1992 at grade 4, there was an increase in the percentage of eighth graders permitted unrestricted use across the nation and in both top and bottom one-third schools, although the change for top one-third schools was not statistically significant. The changes were found in high- and average-ability classrooms, rather than in low- or mixed-ability classrooms.

Table 4.1

Teachers' Reports on Permitting the Use of Calculators in Mathematics Class, Grades 4 and 8

	Assessment Years	UNRESTRICTED USE		RESTRICTED USE	
		Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Grade 4					
Nation	1992	5 (1.1)	219 (5.7)	95 (1.1)	218 (0.9)>
	1990	3 (0.8)	214 (4.2)	97 (0.8)	215 (1.1)
Top One-Third	1992	6 (2.5)	242 (5.1)	94 (2.5)	236 (1.0)>
	1990	4 (1.6)	229 (8.2)	96 (1.6)	229 (1.6)
Bottom One-Third	1992	7 (1.7)	200 (3.8)>	93 (1.7)	196 (1.5)
	1990	4 (1.2)	189 (3.0)	96 (1.2)	196 (1.9)
High Ability	1992	8 (4.7)	247 (6.7)	92 (4.7)	237 (3.7)
	1990	2 (2.2)	—	98 (2.2)	235 (5.6)
Average Ability	1992	5 (1.7)	220 (8.4)	95 (1.7)	222 (1.2)>
	1990	4 (1.6)	230 (7.8)	96 (1.6)	214 (1.7)
Low Ability	1992	4 (2.0)	185 (5.4)	96 (2.0)	196 (2.1)
	1990	3 (1.7)	187 (9.5)	97 (1.7)	203 (3.9)
Mixed Ability	1992	4 (1.3)	215 (5.5)	96 (1.3)	217 (1.2)
	1990	3 (1.4)	207 (7.9)	97 (1.4)	213 (1.7)
Grade 8					
Nation	1992	30 (2.3)>	280 (2.2)	70 (2.3)<	264 (1.3)
	1990	18 (3.1)	280 (3.4)	82 (3.1)	260 (1.5)
Top One-Third	1992	34 (5.2)	295 (4.3)	66 (5.2)	286 (1.5)>
	1990	21 (5.9)	288 (4.6)	79 (5.9)	279 (2.1)
Bottom One-Third	1992	19 (2.6)>	255 (3.4)	81 (2.6)<	245 (1.3)
	1990	4 (1.6)	278(13.7)	96 (1.6)	243 (2.0)
High Ability	1992	50 (4.8)>	305 (2.2)	50 (4.8)<	294 (2.4)>
	1990	26 (4.5)	300 (3.7)	74 (4.5)	284 (3.0)
Average Ability	1992	29 (3.8)>	270 (2.3)	71 (3.8)<	264 (1.5)
	1990	12 (4.2)	268 (4.1)	88 (4.2)	259 (2.2)
Low Ability	1992	16 (3.5)	246 (3.7)<	84 (3.5)	244 (2.1)
	1990	15 (4.1)	270 (4.9)	85 (4.1)	240 (2.9)
Mixed Ability	1992	21 (4.1)	266 (2.5)	79 (4.1)	260 (1.8)
	1990	16 (6.6)	270 (5.6)	84 (6.6)	254 (3.0)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details).

— Sample size insufficient to permit a reliable estimate.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

A second aspect of school practice relates to using calculators during examination settings. Table 4.2 provides data on the percentages of students allowed unrestricted and restricted calculator use during tests. The national 1992 data at grade 4 reflect a pattern similar to that for general classroom usage, with only 5 percent of the students allowed unrestricted usage of calculators on tests. By grade 8, the percentage jumped to 48 percent of the students, a figure higher than that reported for regular classroom usage. Similar to the reports about regular classroom usage, these results represented a significant increase since 1990, primarily at grade 8 and for students in the top one-third of the schools and for students in high- and average-ability classrooms.

Table 4.2

Teachers' Reports on Permitting the Use of Calculators on Mathematics Tests, Grades 4 and 8

		UNRESTRICTED USE		RESTRICTED USE	
	Assessment Years	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Grade 4					
Nation	1992	5 (1.1)>	227 (4.3)	95 (1.1)<	218 (0.9)
	1990	2 (0.8)	231(18.6)	99 (0.8)	215 (1.1)
Top One-Third	1992	7 (2.5)	240 (6.0)	93 (2.5)	236 (1.0)>
	1990	2 (1.8)	264(18.0)	98 (1.8)	228 (1.6)
Bottom One-Third	1992	3 (1.2)	197 (3.4)	97 (1.2)	197 (1.5)
	1990	3 (1.5)	199 (4.6)	97 (1.5)	195 (1.9)
High Ability	1992	8 (5.3)	250 (2.8)	92 (5.3)	237 (2.7)
	1990	8 (6.4)	264(18.0)	92 (6.4)	232 (4.9)
Average Ability	1992	4 (1.7)	223 (6.1)	96 (1.7)	221 (1.2)>
	1990	0 (0.5)	—	100 (0.5)	215 (1.7)
Low Ability	1992	6 (2.6)	206(16.1)	94 (2.6)	195 (1.9)
	1990	3 (1.5)	201 (6.3)	97 (1.5)	203 (4.0)
Mixed Ability	1992	5 (1.7)	222 (4.6)>	95 (1.7)	216 (1.3)
	1990	1 (0.7)	200 (6.3)	99 (0.7)	213 (1.6)
Grade 8					
Nation	1992	48 (3.0)>	276 (1.8)	52 (3.0)<	262 (1.4)
	1990	32 (4.1)	272 (2.8)	68 (4.1)	259 (1.7)
Top One-Third	1992	56 (5.7)>	292 (2.8)	44 (5.7)<	285 (1.3)>
	1990	34 (7.9)	288 (1.7)	66 (7.9)	276 (2.3)
Bottom One-Third	1992	33 (4.5)	248 (3.0)	67 (4.5)	246 (1.4)
	1990	24 (7.4)	250 (4.4)	76 (7.4)	242 (2.7)
High Ability	1992	67 (3.6)>	303 (2.0)>	33 (3.6)<	292 (3.1)
	1990	39 (5.2)	292 (3.2)	61 (5.2)	285 (3.6)
Average Ability	1992	43 (4.3)>	270 (2.0)	57 (4.3)<	262 (1.7)
	1990	23 (4.7)	267 (3.8)	77 (4.7)	258 (2.2)
Low Ability	1992	42 (6.4)	246 (3.4)	58 (6.4)	243 (2.5)
	1990	24 (6.2)	256 (6.2)	76 (6.2)	240 (3.1)
Mixed Ability	1992	40 (4.9)	261 (2.0)	60 (4.9)	261 (2.0)
	1990	41 (9.0)	263 (5.6)	59 (9.0)	252 (4.0)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). When the proportion of students is either 0 percent or 100 percent, the standard error is inestimable. However, percentages 99.5 percent and greater were rounded to 100 percent and percentages 0.5 percent or less were rounded to 0 percent.

— Sample size insufficient to permit a reliable estimate.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Students' Access to Calculators

Increasing calculator usage in the mathematics curriculum requires ensuring that **all** students have access to a calculator for instruction and homework activities. Table 4.3 contains teachers' responses to a question about whether or not students had access to school-owned calculators for mathematics schoolwork. The 1992 results reveal that 59 percent of grade 4 students had access to school-owned calculators, which represented an increase compared to 1990. Fourth graders with access to school-owned calculators had higher average proficiency than their counterparts with no such access. Also, more fourth graders in the top one-third schools than in the bottom one-third schools had access to school-owned calculators.

At grade 8, the question was revised in 1992 to include access to scientific as well as four-function calculators. However, the school-owned calculators to which eighth graders had access were basically four-function calculators. Even though the NCTM standards recommend that students have scientific calculators at this level of instruction, the ratio of four-function to scientific calculators available was about 2 to 1. However at grade 8, their was no relationship between mathematics achievement and access to school-owned calculators — either four-function or scientific.

Table 4.3

Teachers' Reports on Students' Access to School-Owned Calculators for Mathematics Schoolwork, Grades 4 and 8

		YES		NO	
	Assessment Years	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Grade 4					
Nation	1992	59 (3.1)>	221 (1.3)	41 (3.1)<	214 (1.5)
	1990	44 (3.5)	218 (1.8)	56 (3.5)	212 (1.7)
Top One-Third	1992	64 (6.9)	237 (1.6)>	36 (6.9)	234 (1.3)
	1990	48 (7.6)	231 (2.2)	52 (7.6)	228 (2.4)
Bottom One-Third	1992	43 (5.1)	199 (2.5)	57 (5.1)	195 (1.6)
	1990	30 (6.1)	195 (3.2)	70 (6.1)	196 (2.2)
Grade 8 – Basic 4-Function					
Nation	1992	64 (3.0)	269 (1.5)	36 (3.0)	269 (1.8)
Top One-Third	1992	66 (5.2)	287 (2.4)	34 (5.2)	293 (1.8)
Bottom One-Third	1992	62 (4.7)	248 (1.8)	38 (4.7)	247 (1.9)
Grade 8 – Scientific					
Nation	1992	35 (3.0)	272 (2.3)	65 (3.0)	268 (1.3)
Top One-Third	1992	33 (6.1)	292 (3.0)	67 (6.1)	288 (1.5)
Bottom One-Third	1992	30 (4.9)	248 (3.4)	70 (4.9)	247 (1.4)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details).

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

In 1992, students at all three grades were asked whether they had a calculator that they could use to do mathematics schoolwork. As shown in Table 4.4, the percentages of students reporting "yes" rose from 46 percent at grade 4 to 81 percent at grade 8 to 92 percent at grade 12 for students taking mathematics. At grades 8 and 12, those students reporting access to a calculator had higher proficiency than those who did not. More students in top one-third schools than in bottom one-third schools had a calculator to use in doing their mathematics schoolwork.

Among various subgroups of students, White eighth graders were more likely to have such access than either Black or Hispanic students. For twelfth graders enrolled in mathematics classes, there were no significant differences between the percentages of White and Black students having calculators, but a significantly larger percentage of White students had a calculator than did Hispanic students. There were modest gender differences in students' reports about calculator access favoring females, although only the grade 8 difference was statistically significant. Such differences have implications for bringing quality mathematics to all students in our mathematics classrooms.[39]

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[39] Rigol, G., "Balancing Educational, Administrative and Equity Interests in the Development of a Calculator Policy for National Testing Programs." Paper presented at the annual meeting of the National Council on Measurement in Education, Atlanta, GA, April 1993.

Table 4.4

Students' Reports on Having a Calculator to Use for Mathematics Schoolwork, Grades 4, 8, and 12

	YES		NO	
Assessment Year – 1992	**Percentage of Students**	**Average Proficiency**	**Percentage of Students**	**Average Proficiency**
Grade 4				
Nation	46 (1.2)	219 (0.9)	54 (1.2)	218 (0.8)
Top One-Third	53 (2.0)	237 (1.1)	47 (2.0)	237 (1.1)
Bottom One-Third	41 (1.7)	193 (1.8)	59 (1.7)	199 (1.1)
White	45 (1.5)	229 (1.0)	55 (1.5)	226 (1.0)
Black	49 (1.6)	190 (1.5)	51 (1.6)	193 (1.7)
Hispanic	45 (2.2)	200 (1.9)	55 (2.2)	202 (1.6)
Asian/Pacific Islander	47 (3.3)	234 (4.0)	53 (3.3)	229 (3.2)
American Indian	50 (4.9)	212 (4.5)	50 (4.9)	206 (3.6)
Male	44 (1.6)	220 (1.4)	56 (1.6)	220 (0.9)
Female	48 (1.4)	218 (1.2)	52 (1.4)	217 (1.0)
Grade 8				
Nation	81 (1.0)	271 (1.0)	19 (1.0)	256 (1.2)
Top One-Third	88 (1.6)	290 (1.4)	12 (1.6)	285 (2.3)
Bottom One-Third	70 (1.8)	248 (1.1)	30 (1.8)	241 (1.3)
White	84 (1.0)	279 (1.1)	16 (1.0)	270 (1.7)
Black	73 (2.5)	239 (1.5)	27 (2.5)	232 (1.9)
Hispanic	72 (1.6)	251 (1.6)	28 (1.6)	238 (1.8)
Asian/Pacific Islander	91 (2.2)	289 (4.9)	9 (2.2)	276(14.1)
American Indian	71 (6.3)	254 (3.4)	29 (6.3)	255 (4.6)
Male	79 (1.1)	270 (1.3)	21 (1.1)	258 (1.5)
Female	83 (1.0)	271 (1.1)	17 (1.0)	254 (1.6)
Grade 12 – Taking Math†				
Nation	92 (0.5)	309 (0.9)	8 (0.5)	282 (2.4)
Top One-Third	96 (0.9)	323 (1.4)	4 (0.9)	297 (4.0)
Bottom One-Third	86 (1.5)	286 (1.4)	14 (1.5)	270 (3.5)
White	94 (0.6)	315 (1.0)	6 (0.6)	294 (2.7)
Black	89 (1.4)	282 (1.8)	11 (1.4)	267 (4.4)
Hispanic	82 (1.8)	292 (1.9)	18 (1.8)	267 (4.9)
Asian/Pacific Islander	91 (2.8)	322 (3.7)	9 (2.8)	283 (8.2)
American Indian	86 (7.8)	295 (9.6)	14 (7.8)	282(20.4)
Male	91 (0.8)	310 (1.2)	9 (0.8)	283 (3.3)
Female	93 (0.7)	307 (1.1)	7 (0.7)	280 (3.0)

† Sixty-four percent of the twelfth-grade students reported that they were taking a mathematics class at the time of the assessment.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details).

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Frequency of Using Calculators in Mathematics Instruction

Teachers and students were asked to report on the frequency of use of calculators in mathematics class. The data are contained in Tables 4.5 and 4.6. At grades 8 and 12, students who used a calculator at least weekly had higher mathematics proficiency than their counterparts who never or hardly ever used a calculator in mathematics classes.

At grade 4, teachers reported that only 17 percent of the students made weekly use of the calculator in mathematics class. Fifty-one percent were reported as never using the calculator in mathematics class. No significant differences appeared in the degree to which the teachers reported calculator use by ability grouping, nor were there differences in the related mathematical proficiency levels within these groupings. The apparent differences in use between the top one-third and bottom one-third schools did not tend to be statistically significant, although more students in bottom one-third schools (61 compared to 45 percent) were never or hardly ever asked to use the calculator. Fourth graders' reports of calculator usage confirmed the pattern reported by the teachers.

At grade 8, teachers reported that 56 percent of the students used their calculators at least weekly and another 20 percent less than weekly. There were differences among the weekly usage rates for students in classes of differing ability levels, with more students in high-ability classes frequently using calculators than students in low-, average-, or mixed-ability classes. These results reflected significant increases between 1990 and 1992 in weekly use for all grade 8 students, as well as for those in high- and average-ability classrooms. Teachers reported using calculators at least weekly with increased percentages of eighth graders in top one-third schools, but no change was observed in bottom one-third schools. They also reported more frequent calculator use in top one-third schools than in bottom one-third schools. For example, in 1992, teachers reported at least weekly calculator use for 67 percent of the students in higher-performing schools compared to 39 percent in lower-performing schools.

Eighty-two percent of the twelfth graders taking mathematics reported using calculators on a weekly basis, while only 11 percent never or hardly ever used them. The students reporting more frequent calculator use had higher average proficiency. The percentage for weekly use, like those at the other two grades, represented a significant increase from the 75 percent of twelfth graders so reporting in 1990. Use increased significantly in top one-third schools, but not in bottom one-third schools. In 1992, significantly more twelfth graders in top one-third schools than in bottom one-third schools reported using a calculator at least weekly in mathematics class, 86 compared to 73 percent.

Table 4.5

Teachers' Reports on the Frequency of Calculator Use in Mathematics Class, Grades 4 and 8

	Assessment Years	AT LEAST WEEKLY		LESS THAN ONCE A WEEK		NEVER OR HARDLY EVER	
		Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Grade 4							
Nation	1992	17 (2.0)	222 (3.0)	32 (2.0)	221 (1.5)	51 (2.5)	215 (1.3)
	1990	18 (3.2)	219 (2.9)	36 (3.3)	217 (2.0)	46 (3.3)	211 (1.7)
Top One-Third	1992	23 (4.4)	240 (3.6)	32 (4.6)	236 (1.6)	45 (6.1)	233 (1.0)>
	1990	21 (6.6)	233 (3.5)	38 (6.4)	232 (2.6)	41 (7.1)	226 (2.6)
Bottom One-Third	1992	15 (2.8)	193 (3.5)	24 (4.3)	199 (3.1)	61 (4.8)	197 (1.6)
	1990	13 (2.8)	194 (3.9)	26 (5.8)	196 (2.7)	61 (6.6)	195 (2.3)
High Ability	1992	38 (8.7)	246 (6.4)	17 (5.4)	236 (4.9)	46 (8.0)	232 (3.2)
	1990	26 (7.9)	242 (8.2)	40 (9.0)	244 (7.1)	34 (9.8)	223 (7.1)
Average Ability	1992	13 (2.4)	227 (4.2)	33 (3.2)	225 (2.3)>	54 (4.1)	218 (1.6)
	1990	13 (3.7)	217 (6.3)	44 (5.7)	214 (2.7)	43 (5.6)	215 (2.7)
Low Ability	1992	17 (3.5)	189 (7.6)	28 (4.1)	197 (4.7)	55 (4.5)	197 (2.8)
	1990	14 (5.2)	196 (9.0)	29 (7.6)	209 (7.9)	58 (8.6)	199 (4.2)
Mixed Ability	1992	16 (3.2)	216 (3.4)	36 (3.6)	220 (1.6)	48 (2.8)	215 (1.8)
	1990	23 (6.6)	218 (4.0)	30 (4.7)	214 (3.6)	47 (5.9)	209 (2.0)
Grade 8							
Nation	1992	56 (2.8)>	275 (1.4)	20 (2.0)<	258 (2.2)	24 (2.4)	264 (1.9)
	1990	42 (4.3)	269 (2.8)	37 (4.0)	259 (2.2)	20 (3.8)	260 (3.7)
Top One-Third	1992	67 (5.1)>	290 (2.3)	16 (3.1)<	284 (2.7)>	17 (3.2)	288 (3.3)>
	1990	42 (8.2)	291 (3.3)	38 (7.9)	274 (2.3)	21 (6.9)	273 (3.4)
Bottom One-Third	1992	39 (4.4)	250 (2.5)	29 (3.8)<	240 (3.2)	32 (4.8)>	248 (2.3)>
	1990	32 (7.4)	244 (3.9)	52 (8.3)	248 (2.9)	17 (3.9)	231 (4.8)
High Ability	1992	70 (3.7)>	302 (1.9)	11 (2.0)<	299 (4.6)>	19 (3.1)	292 (4.2)
	1990	43 (4.5)	295 (3.0)	37 (5.3)	283 (3.8)	20 (4.7)	282 (7.5)
Average Ability	1992	56 (4.2)>	269 (1.8)	20 (2.7)<	260 (3.1)	24 (3.1)	264 (2.4)
	1990	38 (5.4)	266 (3.8)	40 (5.2)	257 (2.7)	22 (4.7)	256 (5.9)
Low Ability	1992	46 (4.7)	350 (3.0)	34 (6.7)	240 (6.4)	21 (5.0)	238 (3.7)
	1990	35 (6.2)	257 (6.0)	42 (6.4)	244 (4.7)	23 (6.4)	240 (5.7)
Mixed Ability	1992	48 (5.1)	264 (1.9)	21 (4.8)	256 (2.1)	30 (4.2)	260 (3.1)
	1990	48 (9.7)	262 (4.6)	33 (8.1)	248 (4.9)	19 (6.8)	258 (6.2)

>The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

<The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 4.6

Students' Reports on the Frequency of Calculator Use in Mathematics Class, Grades 4, 8, and 12

		AT LEAST WEEKLY		LESS THAN ONCE A WEEK		NEVER OR HARDLY EVER	
	Assessment Years	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Grade 4							
Nation	1992	21 (1.0)	216 (1.8)>	21 (1.2)	227 (1.1)	58 (1.6)	217 (0.8)>
	1990	19 (1.9)	207 (2.5)	17 (1.2)	224 (2.1)	64 (2.4)	213 (1.1)
Top One-Third	1992	23 (1.8)	238 (2.5)	28 (2.7)	238 (1.3)	48 (3.5)	236 (1.0)>
	1990	16 (3.6)	232 (2.3)	26 (2.3)	234 (2.4)	58 (4.1)	227 (1.8)
Bottom One-Third	1992	22 (2.0)	189 (2.7)	12 (0.8)	201 (2.5)	66 (2.4)	198 (1.4)
	1990	24 (2.1)	187 (2.1)	11 (1.7)	200 (4.0)	65 (3.1)	197 (1.9)
Grade 8							
Nation	1992	53 (2.0)>	273 (1.2)>	17 (0.8)	265 (1.5)	30 (1.5)<	260 (1.5)
	1990	40 (2.9)	266 (2.2)	20 (1.2)	265 (1.8)	40 (2.9)	258 (1.3)
Top One-Third	1992	62 (3.7)	291 (1.9)	18 (1.4)	285 (1.8)	20 (2.7)<	288 (2.3)>
	1990	46 (6.1)	284 (2.1)	18 (1.8)	282 (2.3)	36 (5.4)	275 (2.0)
Bottom One-Third	1992	39 (3.1)	248 (1.4)>	18 (1.3)	243 (1.7)	43 (3.1)	244 (1.2)
	1990	30 (3.2)	240 (2.8)	22 (2.4)	251 (3.2)	47 (4.7)	243 (2.3)
Grade 12 – Taking Math†							
Nation	1992	82 (1.1)>	310 (1.0)	7 (0.6)	293 (2.4)<	11 (0.9)<	287 (2.1)
	1990	75 (1.5)	307 (1.5)	9 (0.9)	305 (2.7)	16 (1.3)	288 (3.0)
Top One-Third	1992	86 (2.0)>	324 (1.3)	6 (0.9)<	304 (4.1)	8 (1.4)	304 (4.7)
	1990	77 (3.0)	321 (1.4)	10 (1.6)	313 (4.3)	13 (2.2)	307 (6.1)
Bottom One-Third	1992	73 (2.4)	288 (1.4)	10 (1.3)	276 (3.8)	17 (1.9)	271 (2.7)
	1990	70 (3.1)	285 (1.9)	7 (1.1)	286 (4.0)	23 (3.2)	264 (2.3)

† Sixty-four percent of the twelfth-grade students reported that they were taking a mathematics class at the time of the assessment.

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Knowing When to Use a Calculator

Beyond having a calculator is the importance of knowing how and when to use it in mathematical situations. To assess students' understanding of when to use the calculator, the NAEP assessment asked students to indicate the questions where they made use of the calculator in finding a solution. When students reported using the calculator for at least 65 percent of the questions for which a calculator was deemed appropriate by a group of mathematics educators, and did not use the calculator for more than one item for which calculator use was deemed inappropriate, they were labeled as being in the "high" group of appropriate calculator use. The remaining students were placed in the "other" calculator-usage group. The data are provided in Table 4.7.

Across the grades, the percentages of students appropriately using a calculator were 23 percent at grade 4, 26 percent at grade 8, and 31 percent at grade 12 (significant increases from 1990 at grades 8 and 12). At both grades 8 and 12, students classified in the "high" appropriate-use group had significantly higher mathematical proficiency levels than their classmates.

Consistent with these findings, at grade 4 there was no difference in the percentage of students in the high calculator-use group between top one-third and bottom one-third schools. However, at grades 8 and 12, the differences between students in higher- and lower-performing schools were substantial — 35 compared to 18 percent at grade 8, and 41 compared to 21 percent at grade 12.

Table 4.7

Students' Understanding of When to Use a Calculator in Solving Mathematics Problems, Grades 4, 8, and 12

	HIGH CALCULATOR-USE GROUP†		OTHER GROUP†	
Assessment Year – 1992	**Percentage of Students**	**Average Proficiency**	**Percentage of Students**	**Average Proficiency**
Grade 4				
Nation	23 (0.8)	218 (1.5)	77 (0.8)	218 (0.9)
Top One-Third	22 (1.2)	240 (2.0)	78 (1.2)	236 (1.3)
Bottom One-Third	26 (1.3)	196 (2.4)	74 (1.3)	196 (1.4)
Grade 8				
Nation	26 (0.9)	282 (1.5)	74 (0.9)	262 (1.0)
Top One-Third	35 (1.3)	299 (1.9)	65 (1.3)	284 (1.3)
Bottom One-Third	18 (1.1)	259 (2.4)	82 (1.1)	242 (1.1)
Grade 12				
Nation	31 (1.2)	315 (1.2)	69 (1.2)	291 (0.9)
Top One-Third	41 (2.1)	327 (1.5)	59 (2.1)	308 (1.5)
Bottom One-Third	21 (0.9)	292 (2.5)	79 (0.9)	275 (1.1)

† Students in the "High" group used the calculator for at least 65 percent of the calculator-suitable items and used the calculator for no more than one of the calculator-unsuitable items. Students in the "Other" group used the calculator for less than 65 percent of the calculator-suitable items or used it for more than one of the calculator-unsuitable items.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details).

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Computer Availability

In addition to gathering information about calculators, NAEP asked about the availability of computers, the frequency of their use, and how they were used in school mathematics classrooms. Table 4.8 contains schools' reports on the availability of computers for student use. For the nation, 46 percent of the fourth graders had a computer available in their classroom — a significant increase from the 24 percent reported in 1990 — and 67 percent had access to computers in a computer laboratory. In 1992, at grade 8, 17 percent had availability in their classroom and 71 percent had availability in a laboratory. At grade 12, only 5 percent of the students had computers available at all times in their classroom, while 79 percent had access to them in a computer laboratory. These results reflected essentially no changes between 1990 and 1992 at the two older grades. Although the results did not tend to be statistically significant for classroom availability and computer laboratories, there was a pattern of less availability in the lower one-third performing schools. Principals reported fewer students in situations where computers were available to bring to the classrooms when needed.

The increasing availability of laboratory centers at upper grades makes extra-class assignments and special demonstration lessons possible. The use of such laboratories for dynamic geometry investigation and function graphing software appears to be a trend at the secondary level. At the same time, hand-held, graphing calculators are becoming more competitive with the computer and are increasingly being used as portable laboratories for investigating mathematical situations.[40]

Table 4.9 shows teachers' reports at grades 4 and 8 for a similar question about computer availability. Their views about availability in classrooms correspond to those reported by school administrators. Beyond that, however, teachers were asked about students without access to computers in school. They reported that about one-fifth of the fourth graders and one-fourth of the eighth graders did not have access to school computers. There was little difference in the availability of computers across classes by ability grouping at the two grades. Also, there was little difference in teachers' reports between the top- and bottom- performing one-third of the schools.

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[40] Heid, M. K., & Baylor, T., "Computing Technology." In P. S. Wilson, *Research Ideas for the Classroom: High School Mathematics* (New York, NY: Macmillan, 1993).

Table 4.8

Schools' Reports on the Availability of Computers for Student Use In Mathematics Class, Grades 4, 8, and 12

	Assessment Years	Yes, Computers Available All the Time in Mathematics Classrooms Percentage of Students	Yes, Computers Grouped in A Laboratory Available to Mathematics Classrooms Percentage of Students	Yes, Computers Available to Bring to Classrooms When Needed Percentage of Students
Grade 4				
Nation	1992	46 (3.1)>	67 (3.3)	49 (3.5)
	1990	24 (3.7)	56 (4.7)	53 (4.2)
Top One-Third	1992	56 (4.3)>	73 (4.8)>	57 (6.9)
	1990	26 (7.5)	49 (7.9)	57 (8.5)
Bottom One-Third	1992	34 (4.9)	71 (5.1)	34 (4.9)
	1990	28 (6.2)	59 (8.3)	49 (8.8)
Grade 8				
Nation	1992	17 (2.6)	71 (3.1)	52 (3.5)
	1990	14 (4.1)	57 (5.6)	60 (4.5)
Top One-Third	1992	21 (4.8)	78 (5.2)	61 (6.7)
	1990	19 (9.2)	63 (9.9)	66 (9.9)
Bottom One-Third	1992	15 (5.1)	62 (6.1)	31 (4.2)<
	1990	5 (3.4)	63 (8.6)	52 (8.2)
Grade 12				
Nation	1992	5 (1.2)	79 (2.8)	47 (4.0)<
	1990	9 (3.4)	79 (4.9)	64 (5.1)
Top One-Third	1992	8 (3.0)	88 (4.4)	62 (6.6)
	1990	10 (5.3)	89 (6.2)	65 (8.8)
Bottom One-Third	1992	5 (2.0)	71 (5.5)	35 (6.3)<
	1990	4 (3.4)	77 (8.0)	64 (7.8)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details).

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 4.9

Teachers' Reports on the Availability of Computers for Student Use in Mathematics Class, Grades 4 and 8

		AVAILABLE IN CLASSROOM		DIFFICULT TO ACCESS		NOT AVAILABLE	
	Assessment Years	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Grade 4							
Nation	1992	44 (2.8)>	219 (1.7)	38 (2.5)	220 (1.4)>	18 (2.4)	213 (2.5)
	1990	31 (3.7)	221 (2.1)	46 (3.7)	214 (1.8)	23 (2.7)	207 (2.9)
Top One-Third	1992	50 (4.8)	236 (1.5)	34 (4.2)	238 (2.2)>	16 (3.0)	232 (2.0)
	1990	40 (7.4)	232 (2.4)	42 (6.4)	229 (2.5)	17 (4.2)	225 (4.3)
Bottom One-Third	1992	39 (4.0)>	197 (2.2)	36 (4.7)	198 (2.0)	26 (5.3)	193 (2.7)
	1990	19 (5.2)	198 (3.2)	43 (6.2)	197 (2.4)	38 (7.5)	193 (3.0)
High Ability	1992	44 (9.4)	233 (3.3)	42 (8.4)	241 (5.6)	14 (6.0)	243 (3.2)
	1990	39 (9.6)	244 (6.1)	37(11.0)	224(11.1)	24 (8.6)	234 (9.0)
Average Ability	1992	46 (3.8)>	224 (2.1)	35 (3.3)	221 (2.0)	19 (3.3)	216 (2.3)
	1990	27 (4.4)	221 (3.9)	49 (5.3)	217 (2.6)	24 (4.8)	206 (4.2)
Low Ability	1992	40 (5.8)	198 (4.1)	40 (4.7)	197 (2.5)	21 (4.0)	190 (4.8)
	1990	20 (7.9)	219(10.8)	56 (8.3)	203 (5.8)	24 (6.8)	188 (4.4)
Mixed Ability	1992	42 (4.5)	215 (2.2)	40 (4.3)	221 (1.8)>	18 (3.5)	212 (3.6)
	1990	35 (5.2)	216 (3.1)	43 (5.2)	214 (2.0)	22 (4.1)	206 (3.6)
Grade 8							
Nation	1992	20 (2.0)	265 (2.1)	57 (2.7)	272 (1.7)>	24 (1.9)	264 (1.7)
	1990	22 (3.7)	263 (3.2)	50 (4.2)	262 (2.2)	28 (4.0)	266 (2.6)
Top One-Third	1992	14 (3.4)	286 (2.5)	70 (4.1)	290 (1.9)>	16 (4.1)	286 (4.7)
	1990	23 (7.3)	277 (3.2)	56 (6.6)	281 (2.0)	21 (5.3)	285 (3.8)
Bottom One-Third	1992	18 (3.4)	241 (4.2)	49 (4.2)	249 (1.5)	32 (3.5)	247 (1.7)
	1990	19 (4.1)	242 (5.3)	58 (7.4)	242 (3.0)	23 (5.2)	252 (4.7)
High Ability	1992	17 (2.6)	300 (3.1)	63 (4.2)>	301 (2.2)>	20 (2.8)	296 (3.5)
	1990	19 (5.6)	290 (3.9)	44 (5.7)	284 (4.4)	36 (6.3)	291 (3.9)
Average Ability	1992	21 (2.7)	267 (2.2)	52 (4.0)	267 (2.1)	26 (3.3)	261 (2.4)
	1990	16 (4.2)	257 (4.0)	58 (6.3)	260 (2.8)	27 (5.3)	262 (3.5)
Low Ability	1992	22 (3.9)	235 (5.0)	54 (5.6)	248 (2.4)	24 (3.9)	246 (3.1)
	1990	16 (5.1)	237 (6.8)	52 (6.5)	244 (5.2)	32 (5.9)	246 (4.3)
Mixed Ability	1992	18 (3.1)	255 (3.5)	59 (4.8)	265 (2.0)	23 (4.2)	255 (3.3)
	1990	36 (9.2)	259 (5.3)	42 (7.8)	256 (4.8)	22 (6.1)	253 (4.9)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Computer Use

To provide a context for considering the degree of computer use in mathematics classes, students were asked about how often they used the computer in accomplishing any of their schoolwork. As can be seen in Table 4.10, few students reported daily use of computers for their schoolwork across all curriculum areas. The percentage reporting daily use at grade 12 (17 percent) was significantly higher than that at grades 4 and 8 (6 to 8 percent). Large percentages of students — from 66 percent at grade 4 to 46 percent at grade 12 — reported that they never or hardly ever used computers for their schoolwork.

Student proficiency in relation to degree of computer use differed from grade to grade. At grade 4, students who reported using computers in their schoolwork once or twice a month performed significantly higher than students who reported using computers once or twice a week or never or hardly ever. Students in these latter two usage groups performed significantly higher than students reporting daily usage. At grade 8, the average proficiency of students reporting computer use once or twice a month was significantly higher than the proficiency levels for the other three usage level groups of students. At grade 12, students using the computer once or twice a week performed significantly higher than students using it either daily or once or twice a month. Twelfth graders who reported never or hardly ever using computers in their schoolwork had the lowest average proficiency.

Between the top and bottom one-third schools there was a different pattern at grade 4 than at grades 8 and 12. At grade 4, the frequency of use was similar between students in higher- and lower-performing schools, although more fourth graders in lower one-third schools reported daily use and more fourth graders in higher one-third schools reported monthly use.

At grades 8 and 12, significantly greater percentages of students in top one-third schools than in bottom one-third schools used a computer for their schoolwork at least weekly and significantly more used a computer at least monthly. The percentages never or hardly ever using a computer for schoolwork in top one-third and bottom one-third schools were: 45 compared to 76 percent at grade 8, and 36 compared to 55 percent at grade 12. While the majority of students in top one-third schools made at least monthly use of the computer to help with their schoolwork, the corresponding percentages for students in bottom one-third schools were 24 percent at grade 8 and 45 percent at grade 12.

Table 4.10

Students' Reports on the Frequency of Computer Use for Schoolwork, Grades 4, 8, and 12

	ALMOST EVERY DAY		ONCE OR TWICE A WEEK		ONCE OR TWICE A MONTH		NEVER OR HARDLY EVER	
Assessment Year – 1992	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Grade 4								
Nation	6 (0.6)	203 (2.3)	18 (1.2)	217 (1.6)	10 (0.6)	233 (1.6)	66 (1.4)	218 (0.9)
Top One-Third	3 (0.7)	228 (4.6)	18 (2.5)	237 (1.9)	17 (1.5)	244 (1.9)	62 (3.0)	235 (0.9)
Bottom One-Third	10 (1.3)	185 (2.4)	20 (2.2)	195 (2.3)	6 (0.6)	204 (3.4)	65 (2.3)	198 (1.4)
Grade 8								
Nation	8 (0.7)	270 (2.5)	14 (0.7)	274 (1.9)	20 (0.7)	281 (1.5)	58 (1.2)	265 (0.8)
Top One-Third	8 (1.2)	295 (2.8)	18 (1.7)	295 (2.0)	28 (1.7)	294 (1.7)	45 (3.0)	283 (1.1)
Bottom One-Third	8 (0.9)	244 (3.2)	12 (0.9)	246 (2.5)	14 (1.1)	258 (2.3)	67 (1.5)	247 (0.9)
Grade 12								
Nation	17 (0.8)	300 (1.3)	14 (0.6)	314 (1.6)	23 (0.9)	308 (1.4)	46 (1.2)	291 (0.8)
Top One-Third	14 (1.0)	319 (2.5)	20 (1.2)	327 (1.8)	30 (1.5)	320 (1.4)	36 (2.1)	306 (1.5)
Bottom One-Third	18 (1.5)	283 (2.3)	10 (0.9)	288 (3.4)	17 (1.4)	286 (1.7)	55 (1.8)	276 (1.1)

The standard errors of the estimated percentages appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Tables 4.11 and 4.12 provide results on the frequency of computer use in students' mathematics classes. At grade 4, teachers reported that 56 percent of fourth graders made some weekly use of the computer in their mathematics classrooms (compared to 44 percent in 1990). At grade 8, teachers reported considerably less use, with only 10 percent of the students using the computer at least weekly. There was a significant increase, from 53 to 72 percent, in the percentage of eighth graders whose teachers reported never or hardly ever using computers in their mathematics class.

The reports of fourth-grade students differed considerably from those of their teachers. Only 33 percent of the students indicated that they made use of the computer at least once a week in mathematics class. Fifty-eight percent of the fourth graders reported that they never or hardly ever used the computer in mathematics class, a significant increase from 52 percent in 1990. At grade 8, the computer use reported by students was quite similar to that reported by their teachers.

At grade 12, approximately 70 percent of the students currently taking mathematics reported that they never or hardly ever made use of the computer in their study of mathematics. There was a significant decrease between 1990 and 1992 in the weekly use of computers and an increase in the percentage who reported never or hardly ever using a computer as part of mathematics class.

The relationship between teachers' reports of frequency of computer use and students' average proficiency in mathematics showed no clear pattern at grade 4. At grade 8, students whose teachers reported using computers at least weekly had significantly lower performance than students who used them never or hardly ever in their mathematics classes. Similarly, the students' reports at grades 8 and 12 indicated significantly lower average proficiency for students reporting weekly use of computers in mathematics than for students reporting less than weekly use or that they never or hardly ever used computers.

Table 4.11

Teachers' Reports on the Frequency of Computer Use in Mathematics Class, Grades 4 and 8

	Assessment Years	AT LEAST WEEKLY		LESS THAN ONCE A WEEK		NEVER OR HARDLY EVER	
		Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Grade 4							
Nation	1992	56 (3.0)>	219 (1.3)	20 (1.9)	219 (2.5)	24 (2.6)	215 (2.2)
	1990	44 (3.6)	217 (1.6)	23 (2.6)	219 (2.6)	33 (2.4)	208 (2.0)
Top One-Third	1992	58 (4.6)	236 (1.3)>	20 (4.2)	238 (3.6)	22 (4.4)	233 (2.7)
	1990	48 (5.9)	229 (2.0)	28 (5.3)	231 (3.0)	24 (4.4)	228 (3.7)
Bottom One-Third	1992	54 (5.1)>	199 (1.8)	20 (3.9)	198 (1.9)	26 (5.1)<	192 (1.7)
	1990	32 (5.4)	199 (2.4)	18 (3.7)	193 (4.4)	49 (6.3)	194 (2.2)
High Ability	1992	45 (6.8)	238 (3.3)	24 (8.0)	241(11.1)	31 (6.6)	235 (3.7)>
	1990	47(10.4)	245 (4.5)	26 (9.3)	240 (9.4)	27 (8.7)	216 (5.8)
Average Ability	1992	58 (3.8)>	223 (1.7)	18 (2.8)<	222 (3.3)	24 (3.1)	219 (2.6)
	1990	41 (5.1)	216 (2.2)	31 (4.1)	217 (3.1)	28 (4.2)	211 (3.7)
Low Ability	1992	49 (6.0)	198 (3.3)	14 (3.3)	193 (4.1)	37 (6.8)	194 (3.8)
	1990	52 (8.2)	208 (4.9)	13 (6.0)	198(16.1)	35 (7.5)	194 (4.2)
Mixed Ability	1992	58 (4.2)	217 (1.6)	23 (3.2)	216 (2.7)	19 (3.2)<	215 (2.9)
	1990	45 (5.1)	213 (2.1)	22 (4.9)	218 (3.5)	33 (4.6)	208 (3.1)
Grade 8							
Nation	1992	10 (1.2)	259 (2.8)	18 (1.8)<	267 (2.1)	72 (1.9)>	271 (1.3)
	1990	14 (3.4)	252 (4.7)	33 (4.3)	265 (2.9)	53 (4.0)	265 (2.1)
Top One-Third	1992	9 (2.1)	288 (2.5)	10 (2.0)<	292 (4.1)>	80 (2.9)>	288 (1.8)>
	1990	6 (2.4)	280 (3.7)	42 (7.7)	280 (2.4)	52 (7.6)	281 (2.0)
Bottom One-Third	1992	13 (2.7)	238 (4.0)	15 (2.7)<	246 (3.5)	72 (3.5)>	248 (1.3)
	1990	19 (6.7)	229 (4.3)	34 (7.2)	250 (4.3)	48 (6.2)	245 (3.4)
High Ability	1992	9 (2.1)	293 (6.0)	15 (2.4)<	294 (4.4)	76 (2.7)	301 (2.0)>
	1990	12 (4.9)	286 (4.3)	30 (5.5)	291 (4.5)	58 (6.0)	286 (3.3)
Average Ability	1992	10 (1.8)	258 (3.8)>	22 (2.9)<	265 (2.1)	60 (3.0)	267 (1.6)
	1990	9 (3.1)	240 (5.8)	38 (5.8)	264 (2.5)	53 (5.9)	261 (3.4)
Low Ability	1992	12 (2.4)	225 (3.2)	19 (3.3)	246 (6.1)	69 (3.5)	247 (2.7)
	1990	20 (5.7)	228 (8.7)	26 (6.2)	245 (5.9)	54 (6.5)	249 (4.5)
Mixed Ability	1992	11 (2.1)	257 (5.7)	14 (3.5)<	264 (4.5)	76 (3.3)>	261 (1.8)
	1990	17 (8.0)	252 (7.0)	37 (7.6)	255 (6.4)	46 (7.2)	259 (3.3)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 4.12

Students' Reports on the Frequency of Computer Use in Mathematics Class, Grades 4, 8, and 12

		AT LEAST WEEKLY		LESS THAN ONCE A WEEK		NEVER OR HARDLY EVER	
	Assessment Years	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Grade 4							
Nation	1992	33 (1.1)	215 (1.0)	9 (0.6)	227 (1.7)	58 (1.3)>	220 (0.9)>
	1990	37 (1.9)	213 (1.7)	11 (0.9)	221 (2.5)	52 (2.0)	213 (1.1)
Top One-Third	1992	30 (1.9)<	235 (1.3)	12 (1.1)	241 (2.1)	58 (2.4)>	237 (1.0)>
	1990	40 (3.3)	229 (2.6)	13 (1.8)	236 (3.8)	47 (2.9)	228 (2.0)
Bottom One-Third	1992	37 (1.7)	193 (1.5)	6 (0.6)	199 (3.3)	57 (1.8)	199 (1.4)
	1990	31 (2.6)	188 (2.5)	9 (1.3)	198 (3.9)	60 (2.8)	198 (2.3)
Grade 8							
Nation	1992	16 (0.9)	257 (1.7)>	12 (0.7)	271 (2.0)	73 (1.1)	270 (1.0)>
	1990	16 (1.1)	249 (2.3)	14 (1.2)	268 (2.6)	70 (1.5)	265 (1.3)
Top One-Third	1992	13 (1.5)	284 (1.9)>	13 (1.3)	286 (2.9)	74 (1.8)	290 (1.7)>
	1990	13 (1.5)	271 (3.4)	16 (2.6)	285 (3.2)	71 (2.8)	282 (1.7)
Bottom One-Third	1992	19 (1.5)	236 (1.8)	9 (1.0)	250 (2.5)	71 (1.9)	248 (1.0)
	1990	19 (2.6)	229 (2.8)	13 (1.8)	251 (4.0)	68 (3.0)	247 (1.9)
Grade 12 – Taking Math†							
Nation	1992	14 (0.7)<	299 (1.9)	13 (0.7)	314 (1.9)	72 (1.1)>	307 (1.0)
	1990	19 (1.2)	296 (2.8)	15 (1.4)	306 (2.8)	66 (1.7)	306 (1.3)
Top One-Third	1992	13 (0.9)	317 (3.1)	17 (1.5)	325 (3.1)	69 (1.9)	322 (1.8)
	1990	16 (1.4)	315 (2.9)	17 (2.7)	321 (3.8)	67 (3.5)	319 (2.1)
Bottom One-Third	1992	16 (1.2)	277 (3.5)	11 (1.1)	291 (3.9)	73 (1.5)	285 (1.6)
	1990	20 (2.3)	272 (3.2)	14 (2.1)	281 (3.2)	66 (3.0)	282 (1.9)

† Sixty-four percent of the twelfth-grade students reported that they were taking a mathematics class at the time of the assessment.

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Although neither teachers' nor students' reports about the degree of computer use in mathematics classes showed very much difference between top-performing one-third and bottom-performing one-third schools, the association between more use and lower average proficiency is consistent with research indicating that teachers tend to use the computer for drill and practice with lower-performing students.[41] Therefore, in 1992, NAEP also asked teachers about how they primarily used computers in mathematics instruction. The national results are contained in Table 4.13. Fourth graders' teachers reported never using the computer for 26 percent of the students. For those that did use the computer, the teachers reported using the computer primarily for drill and practice activities with 32 percent of the students, for learning new topics with 2 percent, and for playing learning games with 40 percent. Eighth graders' teachers reported never using the computer for mathematics instruction with the majority of the students — 60 percent. However, computers were used primarily for drill and practice with 23 percent of the eighth graders, for learning new topics with 8 percent, and for working with data with 9 percent.

At grade 4, average proficiency was significantly higher for the students who were asked to use the computer primarily for learning games than it was for the students asked to use the computer primarily for drill and practice. The proficiency of students asked to use the computer primarily to learn new topics was somewhere in between the "learning games" and the "drill and practice" groups. Also, teachers reported using computers primarily for drill and practice for more fourth graders in bottom one-third schools than in top one-third schools, 41 compared to 22 percent. Conversely, they reported using computers primarily for playing learning games for more students in top one-third schools — 54 compared to 28 percent.

At grade 8, there was less difference between the primary uses in top one-third versus bottom one-third schools. However, eighth graders who were asked to use the computer primarily for working with data had higher average proficiency than those asked to use the computer primarily for drill and practice. It does appear that at both grades, teachers were tailoring computer use to the needs of their students and attempting to provide additional help for the lower-performing students. Yet, care should be taken that lower-performing students also have opportunities to use computers for purposes other than drill and practice.

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[41] Becker, H. J., "Instructional Uses of School Computers: Reports from the 1985 National Survey," *Issues, 1-4* (Baltimore, MD: Center for Social Organization of Schools, The Johns Hopkins University, 1986-87).

Table 4.13

Teachers' Reports on the Primary Use of Computers for Instruction, Grades 4 and 8

	DRILL AND PRACTICE		LEARNING NEW TOPICS		PLAYING LEARNING GAMES		DO NOT USE COMPUTERS	
Assessment Year – 1992	**Percent of Students**	**Average Proficiency**	**Percent of Students**	**Average Proficiency**	**Percent of Students**	**Average Proficiency**	**Percent of Students**	**Average Proficiency**
Grade 4								
Nation	32 (2.5)	214 (1.8)	2 (0.7)	219 (7.9)	40 (2.2)	222 (1.5)	26 (2.6)	215 (2.1)
Top One-Third	22 (3.4)	237 (2.8)	2 (1.4)	237 (5.1)	54 (4.3)	235 (1.2)	22 (4.3)	233 (2.0)
Bottom One-Third	41 (5.2)	198 (2.0)	3 (1.1)	196 (5.2)	28 (4.9)	199 (2.4)	28 (5.8)	193 (2.6)
High Ability	15 (4.6)	225 (5.5)	1 (1.4)	207 (8.1)	53 (7.1)	236 (2.9)	30 (7.0)	241 (5.0)
Average Ability	35 (3.6)	218 (2.4)	1 (0.9)	236 (5.7)	40 (3.3)	226 (2.0)	25 (3.4)	218 (2.0)
Low Ability	34 (6.0)	197 (3.6)	4 (2.0)	190 (5.9)	31 (5.4)	200 (5.5)	30 (6.1)	192(13.7)
Mixed Ability	31 (4.1)	213 (2.6)	3 (1.4)	226 (8.7)	42 (4.1)	219 (1.8)	24 (3.8)	213 (3.1)
Grade 8								
Nation	23 (2.3)	264 (2.2)	8 (1.2)	269 (4.1)	9 (1.4)	280 (2.9)	60 (2.6)	269 (1.5)
Top One-Third	23 (4.3)	288 (1.9)	8 (2.3)	290 (5.7)	10 (2.1)	291 (3.9)	59 (5.5)	288 (2.5)
Bottom One-Third	21 (2.9)	241 (3.0)	9 (2.4)	244 (4.9)	2 (1.1)	252 (7.0)	68 (4.1)	249 (1.4)
High Ability	20 (3.3)	294 (4.0)	10 (2.6)	301 (5.4)	8 (2.0)	310 (4.2)	62 (3.9)	300 (2.3)
Average Ability	24 (3.4)	262 (2.2)	8 (1.8)	261 (5.2)	9 (2.5)	276 (4.8)	59 (3.5)	266 (1.8)
Low Ability	29 (4.6)	243 (4.9)	6 (1.8)	222 (3.4)	6 (1.8)	256 (8.1)	59 (4.5)	246 (3.1)
Mixed Ability	18 (3.4)	257 (3.1)	9 (3.0)	268 (4.7)	12 (3.7)	272 (3.6)	62 (4.5)	260 (2.1)

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error. Please note the difference between grades 4 and 8 in the third question.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Summary

While little change in calculator-use policies for mathematics instruction was noted between 1990 and 1992 at grade 4, there was a significant increase in the percentage of eighth graders permitted unrestricted use generally, and in the percentage permitted unrestricted use on tests. These changes were found for the nation and in high- and average-ability classrooms. At grade 8, 29 states showed an increase in the percentage of students permitted unrestricted classroom use and 32 states showed an increase in the percentage of students permitted unrestricted calculator use on tests. Eighth graders permitted unrestricted use had higher average proficiency.

In 1992, more fourth graders had access to school owned four-function calculators than in 1990 (59 compared to 44 percent). At grade 8, 64 percent of the students had access to a four-function calculator, but fewer to a school-owned scientific calculator (35 percent). Eighty-one percent of the eighth graders and 92 percent of the twelfth graders taking mathematics reported having a calculator to use for doing their mathematics schoolwork, but only 46 percent of the fourth graders did.

At grade 4, teachers reported that only 17 percent of the students made weekly use of the calculator in mathematics class. Fifty-one percent were reported as never using the calculator in mathematics class.

At grade 8, teachers reported that 56 percent of the students used their calculators at least weekly and another 20 percent less than weekly. This reflected a significant increase between 1990 and 1992 in weekly use for all grade 8 students, as well as for those in high- and average-ability classrooms. Also, teachers reported that weekly use increased significantly in top one-third schools, but not in bottom one-third schools. In general, students' reports at grades 4 and 8 were similar to those of their teachers. Eighty-two percent of the twelfth graders taking mathematics reported using a calculator on a weekly basis, while only 11 percent reported never or hardly ever using a calculator. They also reported that weekly use of calculators in mathematics class increased between 1990 and 1992 nationally and in top one-third performing schools, but not in bottom one-third schools.

A greater percentage of fourth graders in 1992 than in 1990 had access to a computer in their classroom, 46 compared to 24 percent, and 67 percent had access to computers in a computer laboratory. Teachers reported that for 32 percent of the fourth graders, the primary use of computers for instruction was for drill and practice; for another 40 percent the primary instructional use was playing learning games. Those fourth graders asked primarily to use the computer for drill and practice had lower average proficiency than those asked primarily to use the computer for learning games. These results indicate that teachers may be tailoring computer use to the needs of their students, providing additional help for the less proficient students, although all students can benefit from more challenging computer work.

According to teachers, about one-fourth of the fourth graders were in classes where computers were never or hardly ever used. In comparison, 58 percent of the fourth graders themselves reported never or hardly ever using a computer in mathematics class.

At grade 8, teachers and students were in close agreement — 72 to 73 percent of the students never or hardly ever used a computer in mathematics class. Further, the percentages of eighth graders never or hardly ever using a computer increased. The twelfth graders taking mathematics also reported that 72 percent never used a computer in mathematics class.

5

Students' Perceptions About Mathematics

This chapter contains information about students' perceptions about the value of learning mathematics. This aspect of mathematics education may be considered both as an input and outcome variable, because it is highly related to educational achievement in ways that reinforce higher or lower performance.[42] That is, students who do well in mathematics generally have more positive attitudes about the subject, and those who have more positive attitudes tend to perform better.

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[42] McLeod, D. B., "Research on Affect in Mathematics Education: A Reconceptualization." In D. A. Grouws, *Handbook of Research on Mathematics Teaching and Learning* (New York, NY: MacMillan, 1992).

National Research Council in collaboration with the Committee on Mathematical Sciences in the Year 2000, the Board on Mathematical Sciences, and the Mathematical Sciences Education Board, *Moving Beyond Myths: Revitalizing Undergraduate Mathematics* (Washington, DC: National Academy Press, 1991).

Kober, N., *EDTALK: What We Know About Mathematics Teaching And Learning* (Washington, DC: Council for Educational Development and Research, 1991).

Because of the close interrelationship between attitudes and mathematics achievement, this aspect of mathematics education has achieved considerable prominence. For example, the NCTM standards explain that teaching narrowly defined content in ways that emphasize rote activities can make "children begin to lose their belief that learning mathematics is a sense-making experience."

When discussing the low standing of American students in mathematics compared to that of other countries, Harold Stevenson stated that "the achievement gap is real, that it is persistent, and that it is unlikely to diminish until . . . there are marked changes in the attitudes and beliefs of American . . . students about education."[43] The point is made in *Everybody Counts*, prepared by the National Research Council, that society as a whole needs to understand mathematics has changed, and that members of many occupations are called upon regularly to operate complex machines and to interpret graphs, data, and probabilities.[44] Meanwhile, many high school students are unwilling to study higher mathematics because they are convinced they "just can't do math."[45]

Students' Perceptions About Mathematics

To collect information on students' perceptions of mathematics, students in grades 4, 8, and 12 were asked to complete a series of questions that elicited their level of agreement or disagreement with statements about mathematics.[46] The results for all eight questions are summarized in Figure 5.1 for grades 8 and 12. For each question, students were given five response options from strongly agree to strongly disagree and their answers were averaged. Generally, students had positive attitudes about mathematics.

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[43] Stevenson, H. W., Chen, C., Lee, S., "Mathematics Achievement of Chinese, Japanese, and American Children: Ten Years Later,' *Science*, January 1993.

Stevenson, H. W. & Stigler, J. W., *The Learning Gap, Why Our Schools Are Failing and What We Can Learn from Japanese and Chinese Education* (New York, NY: Summit Books, 1992).

[44] *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*, Lynn Steen, editor, (Washington, DC: National Research Council, National Academy Press, 1989).

[45] Tobias, S., *Succeed with Math, Every Students' Guide to Conquering Math Anxiety* (New York, NY: The College Board, 1987).

[46] Eight questions were asked in total, with five being asked in both 1990 and 1992. The full set of results can be found in the *Data Compendium for the 1992 Mathematics Assessment of the Nation and the States* (Washington, DC: National Center for Education Statistics, U.S. Government Printing Office, 1993).

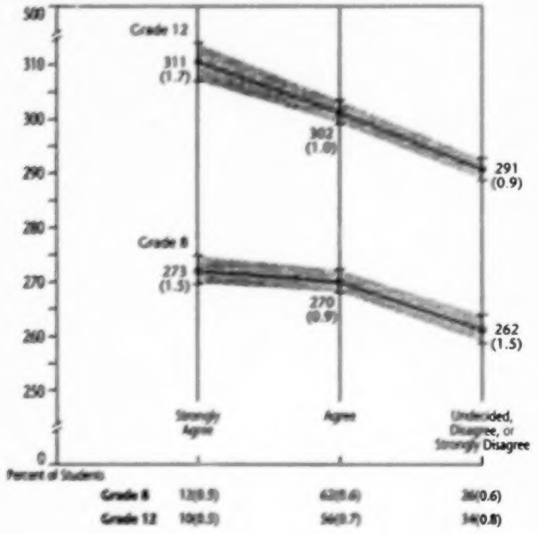
Although only 10 to 12 percent were strongly positive, the majority — 62 percent at grade 8 and 56 percent at grade 12 — agreed, on average, with statements about enjoying mathematics and valuing its utility. Those students with more positive attitudes had higher average mathematics proficiency.

In both 1990 and 1992, students were asked about their level of agreement with the statement "I like mathematics." The data presented in Table 5.1 reveal that students' degree of liking mathematics remained essentially stable across the two-year period. Still, the percentage of twelfth graders in top one-third schools who strongly agreed to liking mathematics decreased from 20 percent in 1990 to 15 percent in 1992. For both 1990 and 1992, a clear positive relationship can be observed between a stronger liking of mathematics and higher achievement. In particular, there was a substantial difference in average proficiency at grades 8 and 12 between students who strongly agreed that they liked mathematics and those who were undecided or disagreed.

The level of agreement with the statement "I like mathematics" decreased from 71 percent at grade four to just about half the twelfth graders (51 percent) who either strongly agreed or agreed. This pattern was consistent across racial/ethnic groups and by gender. While fourth graders were not given the option to strongly agree with the statement, eighth and twelfth graders were, but only 15 to 18 percent did.

Figure 5.1

Average Proficiency by Students' Overall Perceptions of Mathematics



⊥ 95 percent confidence interval.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent confidence for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to the rounding error.

Source: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment.

Table 5.1

Students' Responses to the Statement "I Like Mathematics," Grades 4, 8, and 12

		AGREE		UNDECIDED OR DISAGREE	
	Assessment Years	**Percent of Students**	**Average Proficiency**	**Percent of Students**	**Average Proficiency**
Grade 4					
Nation	1992	71 (0.8)	220 (0.9)>	29 (0.8)	215 (0.9)>
	1990	70 (1.0)	216 (1.1)	30 (1.0)	209 (1.3)
Top One-Third	1992	71 (1.4)	238 (0.9)>	29 (1.4)	233 (1.1)>
	1990	69 (1.7)	232 (2.0)	31 (1.7)	244 (1.9)
Bottom One-Third	1992	72 (1.3)	198 (1.3)	28 (1.3)	192 (1.8)
	1990	68 (1.9)	197 (2.0)	32 (1.9)	189 (1.7)
White	1992	71 (1.0)	229 (1.0)>	29 (1.0)	223 (1.1)>
	1990	69 (1.1)	223 (1.2)	31 (1.1)	215 (1.5)
Black	1992	74 (1.9)	194 (1.5)	26 (1.9)	186 (2.0)
	1990	76 (2.5)	190 (2.0)	24 (2.5)	187 (3.1)
Hispanic	1992	72 (1.7)	203 (1.6)	28 (1.7)	198 (2.4)
	1990	66 (2.3)	204 (2.2)	34 (2.3)	191 (2.7)
Asian/Pacific Islander	1992	80 (2.8)	234 (2.9)	20 (2.8)	222 (4.1)
	1990	74 (4.6)	228 (4.0)	26 (4.6)	226 (9.2)
American Indian	1992	66 (4.5)	209 (3.7)	34 (4.5)	212 (5.3)
	1990	64 (4.4)	214 (4.6)	36 (4.4)	196 (6.9)
Male	1992	72 (1.0)	222 (0.9)>	28 (1.0)	215 (1.4)>
	1990	69 (1.3)	217 (1.4)	31 (1.3)	207 (2.0)
Female	1992	71 (1.0)	219 (1.1)>	29 (1.0)	215 (1.5)
	1990	71 (1.3)	214 (1.3)	29 (1.3)	210 (2.0)

(Table 5.1 continued on the next page)

NOTE: At grade 4, students were not given the "strongly agree" and "strongly disagree" options.

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 5.1

Students' Responses to the Statement "I Like Mathematics," Grades 4, 8, and 12 (continued)

	Assessment Years	STRONGLY AGREE		AGREE		UNDECIDED, DISAGREE, OR STRONGLY DISAGREE	
		Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Grade 8							
Nation	1992	18 (0.6)	275 (1.4)	39 (0.8)	271 (1.1)>	43 (0.9)	263 (1.2)>
	1990	18 (1.1)	271 (2.1)	39 (1.1)	266 (1.6)	43 (1.6)	258 (1.5)
Top One-Third	1992	18 (1.1)	297 (2.0)	40 (2.0)	292 (1.5)>	42 (2.4)	284 (1.4)>
	1990	16 (1.8)	293 (2.0)	40 (1.7)	282 (2.5)	44 (2.8)	275 (2.3)
Bottom One-Third	1992	20 (1.2)	253 (1.9)	38 (0.9)	247 (1.3)	42 (1.2)	242 (1.4)
	1990	21 (2.0)	252 (2.9)	38 (1.8)	246 (2.1)	41 (2.8)	240 (2.5)
White	1992	17 (0.7)	287 (1.6)>	40 (1.0)	280 (1.1)>	44 (1.1)	271 (1.4)>
	1990	16 (1.3)	280 (2.4)	38 (1.4)	274 (1.9)	45 (2.2)	264 (1.6)
Black	1992	26 (1.7)	245 (2.5)	38 (1.7)	236 (1.7)	36 (1.4)	234 (2.3)
	1990	26 (2.3)	252 (4.5)	39 (2.5)	235 (2.5)	35 (2.4)	232 (3.7)
Hispanic	1992	17 (1.5)	260 (2.5)	38 (1.6)	249 (2.0)	45 (2.1)	240 (2.1)
	1990	20 (2.4)	254 (4.7)	39 (2.0)	248 (2.6)	41 (2.6)	238 (2.8)
Asian/Pacific Islander	1992	21 (2.7)	289 (7.4)	44 (2.6)	291 (7.0)	35 (3.1)	286 (5.1)
	1990	20 (5.1)	298 (8.6)	47 (3.2)	279 (4.3)	32 (4.4)	275 (6.0)
American Indian	1992	15 (3.9)	260 (6.7)	36 (4.0)	259 (4.4)	49 (4.6)	249 (3.9)
	1990	17 (3.9)	246(23.6)	27 (5.1)	259 (6.9)	55 (4.9)	237 (9.4)
Male	1992	20 (0.8)	275 (2.0)	40 (1.0)	271 (1.3)	41 (1.0)	262 (1.5)
	1990	20 (1.3)	270 (2.8)	39 (1.5)	267 (1.8)	40 (1.8)	258 (2.2)
Female	1992	16 (0.8)	275 (1.9)	39 (1.0)	271 (1.3)>	45 (1.2)	264 (1.3)>
	1990	16 (1.4)	273 (2.6)	38 (1.5)	264 (2.0)	46 (2.1)	258 (1.3)
Grade 12							
Nation	1992	15 (0.6)	313 (1.6)	36 (0.7)	305 (1.2)>	49 (0.9)	290 (0.9)>
	1990	18 (1.0)	314 (2.1)	36 (1.0)	298 (1.4)	46 (1.4)	284 (1.1)
Top One-Third	1992	15 (1.0)<	334 (2.2)	36 (0.9)	323 (1.4)>	49 (1.2)>	305 (1.2)>
	1990	20 (1.1)	329 (2.1)	36 (1.7)	314 (1.9)	44 (2.1)	298 (1.3)
Bottom One-Third	1992	15 (1.4)	291 (3.0)	35 (1.4)	284 (1.5)>	50 (2.1)	272 (1.2)>
	1990	18 (1.7)	291 (3.3)	36 (1.5)	278 (2.1)	45 (2.2)	265 (2.4)
White	1992	13 (0.5)<	325 (1.6)	36 (0.8)	312 (1.2)>	51 (0.9)>	295 (0.9)>
	1990	17 (1.0)	322 (1.9)	36 (1.2)	305 (1.5)	47 (1.4)	290 (1.2)
Black	1992	21 (1.6)	289 (3.1)	34 (1.7)	278 (2.1)	45 (1.8)	266 (1.9)
	1990	21 (2.0)	284 (4.1)	40 (3.0)	270 (3.4)	39 (3.2)	259 (3.0)
Hispanic	1992	20 (2.6)	291 (3.7)	35 (2.0)	288 (2.4)	45 (4.0)	276 (2.2)>
	1990	16 (2.0)	296 (5.8)	38 (2.3)	285 (3.1)	46 (2.6)	262 (3.5)
Asian/Pacific Islander	1992	25 (3.7)	322 (4.2)	40 (4.5)	318 (4.0)	36 (3.8)	306 (6.1)
	1990	23 (8.0)	324(12.1)	42 (6.3)	320 (5.3)	36 (5.8)	297 (5.4)
American Indian	1992	20 (9.7)	300(20.0)	30 (7.4)	276(10.9)<	50 (8.0)	278(10.8)
	1990	36(14.8)	284(23.3)	22(10.5)	312 (6.4)	41(13.8)	278(12.0)
Male	1992	16 (0.9)	314 (2.6)	37 (1.1)	307 (1.3)	47 (1.0)>	292 (1.3)>
	1990	19 (1.3)	319 (2.5)	39 (1.4)	302 (1.9)	42 (1.7)	284 (1.5)
Female	1992	14 (0.8)	312 (1.9)	34 (1.0)	304 (1.5)>	51 (1.2)	289 (1.0)
	1990	17 (1.2)	308 (2.6)	34 (1.5)	295 (2.0)	49 (1.8)	285 (1.5)

Students' View of the Utility of Mathematics

One indicator of the value students place on mathematics is how they perceive its relevance to daily life. Accordingly, students were asked to state their level of agreement with the statement "Mathematics is useful for solving everyday problems." The data in Table 5.2 show that, in 1992, about two-thirds of fourth graders, four-fifths of eighth graders, and two-thirds of twelfth graders agreed or strongly agreed with the statement. The percentage of eighth graders strongly agreeing with the statement rose from 32 percent in 1990 to 38 percent in 1992. This increase was reflected for students in the top one-third of the schools as well as the bottom one-third of the schools, and for eighth-grade males as well as females.

Fourth graders agreeing with the statement had higher mathematics performance than those who were undecided or disagreed. This was true for not only the nation, but also for males and females, and for all race/ethnicity categories except American Indian. Eighth and twelfth graders were given "strongly agree" or "strongly disagree" response categories; however, those who strongly agreed with the statement did not have greater mathematics proficiency than those who agreed.

Table 5.2

Students' Responses to the Statement "Mathematics Is Useful for Solving Everyday Problems," Grades 4, 8, and 12

		AGREE		UNDECIDED OR DISAGREE	
	Assessment Years	**Percent of Students**	**Average Proficiency**	**Percent of Students**	**Average Proficiency**
Grade 4					
Nation	1992	66 (1.0)	223 (0.8)>	34 (1.0)	214 (1.1)>
	1990	63 (1.1)	216 (1.3)	37 (1.1)	209 (1.2)
Top One-Third	1992	71 (1.6)	239 (1.1)>	29 (1.6)	233 (1.1)>
	1990	68 (2.1)	231 (1.9)	32 (2.1)	226 (2.1)
Bottom One-Third	1992	62 (1.3)	201 (1.4)	38 (1.3)	193 (1.6)
	1990	58 (1.7)	198 (2.2)	42 (1.7)	191 (1.9)
White	1992	67 (1.3)	230 (0.9)>	33 (1.3)	222 (1.2)>
	1990	65 (1.3)	223 (1.4)	35 (1.3)	216 (1.6)
Black	1992	63 (1.8)	196 (1.4)	37 (1.8)	189 (2.0)
	1990	60 (2.8)	190 (2.2)	40 (2.8)	189 (2.4)
Hispanic	1992	61 (1.8)	206 (1.6)	39 (1.8)	196 (2.2)
	1990	58 (2.8)	203 (2.2)	42 (2.8)	195 (2.4)
Asian/Pacific Islander	1992	71 (3.1)	236 (3.1)	29 (3.1)	226 (4.0)
	1990	65 (5.3)	230 (4.8)	35 (5.3)	223 (6.7)
American Indian	1992	68 (4.1)	210 (3.9)	32 (4.1)	210 (5.0)
	1990	63 (5.9)	211 (4.2)	37 (5.9)	202 (6.7)
Male	1992	67 (1.1)	225 (0.9)>	33 (1.1)	214 (1.5)
	1990	64 (1.3)	217 (1.6)	36 (1.3)	209 (1.7)
Female	1992	66 (1.3)	221 (1.1)>	34 (1.3)	214 (1.2)
	1990	62 (1.7)	216 (1.4)	38 (1.7)	209 (1.8)

(Table 5.2 continued on the next page)

NOTE: At grade 4, students were not given the "strongly agree" and "strongly disagree" options.

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 5.2

Students' Responses to the Statement "Mathematics Is Useful for Solving Everyday Problems," Grades 4, 8, and 12 (continued)

		STRONGLY AGREE		AGREE		UNDECIDED, DISAGREE, OR STRONGLY DISAGREE	
	Assessment Years	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Grade 8							
Nation	1992	38 (0.7)>	269 (1.1)>	43 (0.7)	271 (1.1)	19 (0.6)<	264 (1.4)>
	1990	32 (1.0)	263 (1.7)	44 (1.0)	268 (1.5)	24 (1.1)	256 (2.1)
Top One-Third	1992	38 (1.5)>	289 (1.8)>	44 (1.5)	291 (1.3)>	18 (1.2)	287 (2.3)>
	1990	30 (1.8)	281 (2.0)	48 (1.6)	284 (1.7)	23 (1.6)	274 (2.9)
Bottom One-Third	1992	39 (1.2)>	249 (1.2)	41 (1.0)	248 (1.6)	20 (0.9)	242 (1.8)
	1990	34 (1.8)	246 (2.4)	42 (1.9)	249 (2.4)	24 (2.2)	237 (2.8)
White	1992	37 (0.9)>	279 (1.4)>	44 (0.9)	279 (1.1)	20 (0.7)<	273 (1.5)>
	1990	31 (1.1)	271 (1.8)	46 (1.0)	275 (1.7)	24 (1.3)	263 (2.5)
Black	1992	43 (1.8)	240 (1.5)	39 (1.4)	239 (2.0)	18 (1.3)<	234 (3.1)
	1990	38 (2.6)	239 (3.8)	38 (3.0)	243 (3.9)	25 (2.3)	233 (4.4)
Hispanic	1992	37 (1.8)	250 (2.2)	43 (1.5)	251 (1.8)	19 (1.2)<	242 (2.9)
	1990	34 (2.6)	248 (3.9)	40 (3.3)	249 (2.5)	26 (2.3)	240 (4.0)
Asian/Pacific Islander	1992	40 (3.2)	288 (7.0)	43 (3.9)	289 (5.4)	18 (2.8)	295 (7.3)
	1990	33 (5.7)	290 (5.6)	47 (7.1)	277 (5.6)	20 (6.1)	278 (6.7)
American Indian	1992	38 (5.2)	254 (5.1)	40 (4.5)	254 (4.8)	22 (5.0)	255 (4.0)
	1990	42 (7.7)	251 (9.4)	34(13.9)	251 (9.3)	23 (8.3)	237(13.4)
Male	1992	40 (1.0)>	271 (1.6)	42 (0.9)	271 (1.3)	19 (0.8)	262 (2.2)
	1990	35 (1.3)	266 (1.8)	43 (1.4)	268 (2.0)	22 (1.6)	254 (2.9)
Female	1992	36 (0.8)>	268 (1.2)>	44 (0.9)	271 (1.4)	20 (0.8)<	267 (1.8)>
	1990	29 (1.5)	259 (2.2)	44 (1.3)	268 (1.8)	26 (1.1)	258 (2.2)
Grade 12							
Nation	1992	24 (0.6)	300 (1.3)	47 (0.6)	301 (1.0)	30 (0.6)	295 (1.1)>
	1990	24 (0.8)	299 (2.0)	49 (1.2)	298 (1.5)	28 (1.1)	288 (1.4)
Top One-Third	1992	22 (1.2)	320 (2.0)	46 (1.1)<	319 (1.3)>	32 (1.2)>	309 (1.6)>
	1990	23 (1.2)	317 (3.7)	52 (1.3)	312 (1.5)	25 (1.4)	300 (2.2)
Bottom One-Third	1992	26 (1.0)	278 (2.1)	47 (1.0)	282 (1.4)	26 (1.1)	277 (1.7)>
	1990	28 (2.0)	278 (3.0)	46 (2.5)	278 (2.0)	27 (1.8)	267 (3.0)
White	1992	21 (0.8)	310 (1.6)	47 (0.8)	307 (1.1)	31 (0.7)	299 (1.1)>
	1990	21 (1.0)	309 (2.2)	51 (1.4)	302 (1.6)	28 (1.2)	293 (1.6)
Black	1992	31 (1.5)	277 (2.6)	45 (1.5)	276 (2.1)	24 (1.4)	271 (2.6)
	1990	34 (2.3)	270 (3.6)	43 (2.4)	269 (3.4)	23 (2.0)	266 (3.0)
Hispanic	1992	27 (1.8)	281 (2.5)	46 (2.2)	288 (2.3)	27 (1.8)	282 (2.7)>
	1990	28 (2.9)	276 (4.6)	41 (3.6)	287 (3.3)	31 (3.2)	264 (5.3)
Asian/Pacific Islander	1992	26 (2.2)	317 (4.5)	50 (3.7)	318 (4.3)	24 (2.7)	309 (4.7)
	1990	31 (4.6)	310(12.6)	49 (6.3)	313 (5.2)	20 (4.1)	321 (9.1)
American Indian	1992	30(12.4)	283 (8.8)	36 (9.8)	281(12.3)<	34 (7.1)	284(18.6)
	1990	29(11.7)	271(24.8)	41(15.2)	315 (6.2)	30(11.7)	267(15.3)
Male	1992	26 (0.9)	302 (1.6)	46 (1.0)	304 (1.3)	28 (0.8)	295 (1.5)>
	1990	28 (1.3)	304 (2.7)	46 (1.6)	302 (1.9)	26 (1.5)	287 (2.0)
Female	1992	21 (0.7)	298 (1.7)	48 (1.0)	299 (1.2)	31 (0.9)	294 (1.4)>
	1990	20 (1.0)	292 (2.4)	52 (1.3)	295 (2.1)	29 (1.2)	288 (1.6)

Another indicator of mathematics' value to students is its perceived utility in jobs. Table 5.3 enumerates the level of agreement by grade with the statement "Almost all people use mathematics in their jobs." Seventy-four percent of fourth graders agreed with this statement, increasing from 64 percent in 1990. There were increases from 1990 to 1992 in the percentages of fourth graders agreeing with this statement in both the top and bottom one-third schools and across both genders and all race/ethnicity categories (although the change for American Indian students was not statistically significant).

For the eighth grade, 87 percent of the students strongly agreed or agreed with the statement, which was a significant increase from 82 percent in 1990. From 1990 to 1992, there was an increase in the percentage of eighth graders strongly agreeing with this statement among the top- and bottom-performing schools, both genders, and all race/ethnicity categories except Asian/Pacific Islander and American Indian students (where the apparent increases were not statistically significant).

In 1992, 74 percent of twelfth graders agreed or strongly agreed with the statement, which represented essentially no change from 1990 (76 percent). However, compared to 1990, the percentage who strongly agreed with the statement was higher for the nation, for White, Black, and Hispanic students, and for males and females, but these increases were mainly due to decreases in the percentage of those agreeing, and not those ambivalent or disagreeing. The percentage of twelfth-grade Asian/Pacific Islander students expressing ambivalence or disagreement with the statement increased from 9 percent in 1990 to 19 percent in 1992.

Table 5.3

Students' Responses to the Statement "Almost All People Use Mathematics in Their Jobs," Grades 4, 8, and 12

		AGREE		UNDECIDED OR DISAGREE	
	Assessment Years	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Grade 4					
Nation	1992	74 (0.9)>	223 (0.8)>	26 (0.9)<	212 (1.2)>
	1990	64 (1.4)	217 (1.1)	36 (1.4)	208 (1.3)
Top One-Third	1992	78 (1.2)>	238 (1.0)>	22 (1.2)<	233 (1.6)>
	1990	66 (3.0)	232 (1.4)	34 (3.0)	226 (2.0)
Bottom One-Third	1992	68 (1.2)>	201 (1.2)	32 (1.2)<	192 (1.6)
	1990	60 (1.8)	198 (2.1)	40 (1.8)	190 (2.0)
White	1992	76 (1.0)>	230 (0.9)>	24 (1.0)<	222 (1.4)>
	1990	67 (1.6)	222 (1.3)	33 (1.6)	217 (1.7)
Black	1992	70 (1.8)>	196 (1.4)	30 (1.8)<	186 (2.5)
	1990	57 (2.9)	192 (2.3)	43 (2.9)	186 (2.0)
Hispanic	1992	67 (1.7)>	206 (1.4)	33 (1.7)<	193 (2.2)
	1990	58 (2.5)	205 (2.1)	42 (2.5)	192 (3.2)
Asian/Pacific Islander	1992	69 (3.6)>	238 (2.6)	31 (3.6)<	221 (3.0)
	1990	52 (6.1)	231 (5.0)	48 (6.1)	224 (6.5)
American Indian	1992	78 (3.8)	212 (3.1)	22 (3.8)	206 (4.9)
	1990	69 (5.1)	215 (4.0)	31 (5.1)	192 (6.6)
Male	1992	74 (1.0)>	224 (0.9)>	26 (1.0)<	214 (1.4)>
	1990	65 (1.7)	218 (1.6)	35 (1.7)	208 (1.7)
Female	1992	74 (1.0)>	221 (1.1)>	26 (1.0)<	211 (1.5)
	1990	63 (1.8)	216 (1.1)	37 (1.8)	208 (1.8)

(Table 5.3 continued on the next page)

NOTE: At grade 4, students were not given the "strongly agree" and "strongly disagree" options.

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 5.3

Students' Responses to the Statement "Almost All People Use Mathematics in Their Jobs," Grades 4, 8, and 12 (continued)

		STRONGLY AGREE		AGREE		UNDECIDED, DISAGREE, OR STRONGLY DISAGREE	
	Assessment Years	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Grade 8							
Nation	1992	45 (0.9)>	269 (1.0)>	42 (0.7)<	270 (1.1)>	13 (0.6)<	266 (1.7)>
	1990	32 (1.1)	262 (1.8)	50 (1.0)	266 (1.4)	18 (1.1)	259 (2.2)
Top One-Third	1992	44 (2.3)>	289 (2.0)>	45 (1.7)<	290 (1.2)>	11 (1.1)	288 (3.0)
	1990	30 (2.1)	279 (2.0)	54 (2.1)	282 (1.6)	17 (2.1)	278 (3.6)
Bottom One-Third	1992	47 (1.3)>	249 (1.2)	42 (1.1)	246 (1.7)	11 (0.7)<	244 (2.2)
	1990	35 (2.1)	246 (3.0)	46 (2.0)	246 (2.0)	18 (2.0)	240 (1.9)
White	1992	44 (1.2)>	278 (1.2)>	43 (0.9)<	279 (1.2)>	13 (0.7)<	274 (1.7)>
	1990	30 (1.3)	270 (2.1)	51 (1.3)	273 (1.4)	19 (1.4)	265 (2.6)
Black	1992	50 (1.8)>	240 (1.6)	40 (1.8)	238 (2.5)	10 (1.0)<	234 (4.4)
	1990	38 (2.6)	236 (4.0)	46 (2.4)	242 (3.1)	16 (1.9)	234 (3.7)
Hispanic	1992	46 (1.5)>	250 (1.5)	41 (1.6)<	249 (1.8)	13 (1.3)<	240 (5.1)
	1990	31 (2.6)	250 (3.0)	50 (2.3)	245 (3.1)	19 (2.1)	241 (4.3)
Asian/Pacific Islander	1992	52 (3.3)	287 (6.6)	38 (3.3)	293 (5.3)	10 (1.9)	290 (5.9)
	1990	39 (4.9)	284 (6.6)	48 (3.4)	280 (5.3)	13 (3.4)	279 (9.1)
American Indian	1992	47 (4.5)	254 (4.1)	40 (3.5)	258 (3.8)	12 (2.9)	249 (6.0)
	1990	42 (4.7)	249 (7.2)	46(11.2)	243(12.9)	12 (9.5)	257(11.1)
Male	1992	47 (1.2)>	269 (1.3)	40 (0.9)<	271 (1.5)	13 (0.7)<	263 (2.8)
	1990	34 (1.6)	265 (2.4)	48 (1.4)	266 (2.0)	17 (1.3)	257 (3.3)
Female	1992	44 (1.0)>	268 (1.2)>	44 (0.9)<	270 (1.4)	12 (0.7)<	269 (2.0)>
	1990	30 (1.6)	259 (1.8)	51 (1.6)	265 (1.5)	19 (1.5)	260 (2.5)
Grade 12							
Nation	1992	25 (0.6)>	297 (1.3)	49 (0.6)<	300 (1.0)	26 (0.6)	299 (1.1)>
	1990	20 (1.0)	295 (2.1)	56 (1.4)	296 (1.3)	24 (1.1)	293 (1.6)
Top One-Third	1992	22 (1.2)>	317 (1.7)	48 (1.3)<	317 (1.2)>	30 (1.4)	314 (1.7)>
	1990	18 (1.1)	313 (3.4)	55 (1.9)	312 (1.6)	27 (1.9)	306 (2.2)
Bottom One-Third	1992	31 (1.4)>	277 (1.7)	49 (1.3)<	281 (1.2)	20 (1.3)	280 (2.0)>
	1990	24 (2.2)	275 (2.6)	56 (2.3)	277 (1.5)	20 (1.3)	267 (3.6)
White	1992	20 (0.7)>	306 (1.5)	50 (0.7)<	306 (1.0)	28 (0.7)	304 (1.2)>
	1990	18 (1.2)	304 (2.4)	56 (1.5)	301 (1.6)	26 (1.4)	298 (1.6)
Black	1992	34 (1.3)>	274 (2.6)	47 (1.6)	275 (2.2)	19 (1.4)	277 (2.5)>
	1990	27 (2.7)	266 (3.8)	52 (2.3)	272 (2.6)	21 (1.9)	264 (4.0)
Hispanic	1992	33 (2.0)>	282 (2.3)	44 (2.2)<	286 (2.1)	23 (2.2)	283 (3.9)
	1990	20 (2.1)	278 (4.9)	60 (2.2)	278 (3.3)	20 (2.6)	269 (6.8)
Asian/Pacific Islander	1992	32 (3.2)	316 (4.5)	50 (2.7)	313 (4.2)	19 (2.6)>	320 (5.9)
	1990	26 (7.0)	312(10.6)	65 (7.6)	312 (7.1)	9 (2.9)	320 (7.1)
American Indian	1992	23(11.1)	287(10.0)	44 (8.3)	272 (9.9)	33 (7.0)	291(18.7)
	1990	16 (5.1)	274(20.7)	53(13.2)	289(15.4)	31(12.3)	293(12.6)
Male	1992	27 (0.9)>	300 (1.7)	49 (0.9)<	303 (1.3)	25 (0.8)	299 (1.8)>
	1990	20 (1.2)	304 (2.9)	57 (1.8)	299 (1.7)	23 (1.4)	292 (2.1)
Female	1992	24 (0.7)>	294 (1.7)	49 (1.0)<	297 (1.2)	27 (0.8)	300 (1.5)>
	1990	19 (1.2)	287 (2.6)	55 (1.3)	294 (1.8)	25 (1.5)	293 (1.9)

Students' Motivation on NAEP

The motivation of students to work hard on assessments that do not affect their educational record has been a topic of study for NAEP. There has been research indicating that students do perform better if they are highly motivated.[47] The studies that were designed by NAEP's Technical Review Panel to explore this issue have been less conclusive, essentially suggesting little or no difference in performance for most populations of students across varying degrees of motivational settings.[48] As part of studying this motivational issue in the context of NAEP, the 1992 assessment presented students with five questions after they completed the assessment. Table 5.4 shows the results to the question asking how important it was for students to perform well on the NAEP Mathematics Test. The importance of performing well on NAEP declined dramatically from fourth to twelfth grade, with 66 percent stating it was very important at grade 4 and only 9 percent so stating at grade 12. However, students' ratings of importance were not positively related to their average mathematics achievement. Although patterns differed across grades and subpopulations, students who reported that it was very important for them to perform well on NAEP did not have the highest average proficiency compared to students falling in other categories.

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[47] Madaus, G. F., et al, *The Influence of Testing on Teaching Math and Science in Grades 4-12* (Boston, MA: Center for the Study of Testing, Evaluation, and Educational Policy, Boston College, 1992).

[48] Linn, R. L., *Raising the Stakes of Test Administration: The Impact on Student Performance on NAEP* (U.S. Department of Education, National Center for Education Statistics, 1993).

O'Neil, H. F., Jr., *Experimental Studies on Motivation and NAEP Test Performance* (U.S. Department of Education, National Center for Education Statistics, 1992).

The results to the full set of questions, which also asked about how hard students tried, how well they thought they did, and how the assessment compared to their normal school tests can be found in the *Data Compendium for the NAEP 1992 Mathematics Assessment of the Nation and the States* (Washington, DC: National Center for Education Statistics, U.S. Government Printing Office, 1993).

Table 5.4

Students' Reports on How Important It Was for Them to Perform Well on the NAEP Mathematics Test, Grades 4, 8, and 12

Assessment Year – 1992	VERY IMPORTANT		IMPORTANT		SOMEWHAT IMPORTANT		NOT VERY IMPORTANT	
	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Grade 4								
Nation	66 (0.9)	217 (0.9)	23 (0.7)	225 (1.0)	7 (0.4)	222 (1.9)	4 (0.4)	213 (3.1)
Top One-Third	59 (1.8)	236 (1.2)	28 (1.2)	238 (1.2)	8 (1.0)	238 (2.2)	4 (0.7)	239 (3.6)
Bottom One-Third	72 (1.4)	197 (1.3)	18 (1.1)	201 (1.8)	5 (0.5)	193 (3.1)	4 (0.5)	185 (5.4)
White	63 (1.9)	226 (1.0)	26 (0.8)	230 (1.1)	8 (0.5)	228 (1.8)	4 (0.4)	224 (3.3)
Black	77 (1.2)	192 (1.4)	14 (1.0)	196 (2.7)	5 (0.7)	193 (5.7)	4 (0.7)	179 (5.9)
Hispanic	68 (1.9)	201 (1.6)	22 (1.4)	205 (2.6)	6 (0.7)	196 (6.1)	4 (0.8)	193 (6.5)
Asian/Pacific Islander	65 (3.5)	228 (3.3)	24 (2.7)	239 (2.7)	8 (1.9)	245 (8.8)	4 (0.9)	219 (7.9)
American Indian	69 (4.3)	208 (3.6)	19 (4.5)	213 (8.0)	6 (2.1)	205 (8.9)	6 (1.5)	220 (9.0)
Male	64 (1.1)	218 (0.9)	23 (0.9)	226 (1.5)	7 (0.5)	221 (2.3)	5 (0.6)	218 (3.5)
Female	67 (1.1)	215 (1.1)	23 (0.8)	224 (1.5)	6 (0.6)	222 (2.4)	3 (0.3)	204 (4.9)
Grade 8								
Nation	26 (0.8)	261 (1.2)	34 (0.9)	270 (1.2)	27 (0.6)	271 (1.3)	13 (0.6)	270 (1.6)
Top One-Third	22 (1.5)	288 (1.6)	35 (1.5)	290 (1.4)	29 (1.2)	289 (2.7)	14 (0.9)	290 (2.9)
Bottom One-Third	35 (1.7)	242 (1.2)	34 (1.1)	247 (1.2)	21 (0.9)	249 (1.9)	9 (0.7)	246 (3.0)
White	22 (0.8)	275 (1.5)	34 (1.2)	279 (1.2)	30 (0.8)	277 (1.5)	14 (0.7)	277 (1.8)
Black	39 (2.5)	234 (1.8)	33 (1.9)	239 (2.1)	19 (1.3)	241 (2.4)	9 (1.2)	237 (4.6)
Hispanic	35 (2.2)	242 (1.9)	33 (1.6)	248 (1.6)	22 (2.0)	251 (2.5)	10 (1.3)	246 (5.0)
Asian/Pacific Islander	23 (2.6)	282 (8.4)	35 (4.2)	293 (9.6)	27 (3.4)	286 (4.1)	15 (2.4)	290 (5.2)
American Indian	21 (3.8)	257 (6.0)	40 (3.7)	258 (4.2)	20 (2.9)	244 (5.2)	19 (3.0)	254 (6.6)
Male	26 (0.9)	263 (1.7)	31 (1.0)	269 (1.6)	27 (0.6)	270 (1.5)	16 (0.6)	269 (2.0)
Female	26 (1.0)	259 (1.4)	37 (1.2)	271 (1.5)	27 (1.1)	271 (1.9)	10 (0.7)	272 (2.6)
Grade 12								
Nation	9 (0.5)	292 (1.7)	25 (0.7)	298 (1.3)	36 (0.7)	300 (1.0)	31 (0.9)	300 (1.2)
Top One-Third	6 (0.6)	317 (3.7)	22 (1.2)	320 (2.0)	37 (1.3)	316 (1.3)	36 (1.7)	314 (1.5)
Bottom One-Third	12 (1.3)	274 (2.9)	28 (1.5)	278 (1.5)	35 (1.3)	281 (1.5)	24 (1.7)	280 (1.9)
White	6 (0.4)	305 (2.5)	23 (0.7)	307 (1.5)	37 (0.8)	306 (1.0)	34 (1.0)	303 (1.3)
Black	15 (1.4)	275 (3.2)	27 (1.9)	275 (2.3)	36 (2.2)	273 (2.3)	22 (1.7)	278 (3.1)
Hispanic	16 (3.3)	274 (6.0)	34 (2.3)	279 (3.3)	30 (1.8)	290 (2.4)	19 (4.1)	287 (3.6)
Asian/Pacific Islander	15 (2.2)	304 (4.9)	24 (3.1)	317 (6.7)	31 (3.1)	313 (5.1)	30 (3.9)	324 (3.9)
American Indian	11 (7.7)	267(16.3)	29 (6.8)	283(10.0)	28 (9.5)	293 (6.7)	32 (6.6)	276(18.5)
Male	9 (0.7)	295 (2.3)	23 (1.0)	300 (1.9)	33 (1.0)	302 (1.4)	35 (1.1)	302 (1.5)
Female	8 (0.6)	288 (2.3)	26 (0.9)	297 (1.8)	38 (1.0)	299 (1.1)	27 (1.1)	298 (1.5)

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Summary

In both the 1990 and 1992 assessments, more positive responses about liking and valuing mathematics were related to higher average mathematics proficiency. Also, greater percentages of students in 1992 than 1990 reported seeing the utility of mathematics. Although the percentages of students reporting that they liked mathematics remained stable between the two assessments, there were some increases in the percentages of students agreeing that mathematics is useful for solving everyday problems and that almost all people use mathematics in their jobs.

Unfortunately, as students proceeded through their schooling their enjoyment of mathematics declined. In 1992, 71 percent of the fourth graders agreed to liking mathematics compared to 57 percent of the eighth graders and 51 percent of the twelfth graders. Similarly, students reported that the importance of performing well on NAEP declined dramatically from fourth to twelfth grade, with 66 percent stating it was very important at grade 4 and only 9 percent so stating at grade 12. However, in contrast to their opinions about mathematics, students' ratings of importance were not associated with their average mathematics achievement.

6

Students' Mathematics Course Taking

Students who do well in mathematics are more likely to be placed into more advanced classes earlier in their school careers, enabling them to thereby pursue this coursework into high school, most likely learning more mathematics and having higher achievement. Conversely, students who do not do as well in mathematics may become involved in a downward spiral in which they take less advanced coursework or even discontinue their study of mathematics and, therefore, have lower achievement.

Because students' primary opportunity to learn mathematics occurs during their schooling, there has been considerable concern about the amount and kinds of mathematics covered in the school curriculum, and about students' propensity to opt out of taking advanced mathematics coursework. This section and the following section about students' coursework in high school contain the NAEP data on eighth- and twelfth-grade course taking, looking in particular at what point (if ever) students progress from general mathematics courses into more advanced content areas such as algebra and geometry.

Students' Mathematics Coursework In Eighth Grade

Table 6.1 contains eighth graders' reports on the type of mathematics course they were taking at the time of the assessment. Across the nation in 1992, nearly half (49 percent) of the students were taking a general eighth-grade mathematics course, with another 28 percent enrolled in pre-algebra and 20 percent in algebra. Compared to 1990 this represented a significant decrease in the percentage of students enrolled in eighth-grade mathematics and an increase in the percentage enrolled in pre-algebra.

In 1992, students in top one-third schools were more than twice as likely as those in bottom one-third schools to be enrolled in algebra by the eighth grade (27 versus 13 percent). Sixty-one percent of the eighth graders in bottom-third schools were in a general mathematics curriculum rather than pre-algebra or algebra, as opposed to 38 percent of the students in top-third schools.

For the nation and school subgroups, eighth graders who had advanced to an algebra course had consistently higher average proficiencies than students enrolled in pre-algebra, who in turn had higher proficiencies than students taking general mathematics courses. This pattern of higher achievement for students in more advanced courses also appears in Table 6.2, where eighth graders reported on which type of class they planned to take in ninth grade. Significantly more eighth graders in top one-third schools than bottom one-third schools planned to take geometry (19 versus 11 percent) or algebra (44 versus 27 percent), with more students in bottom-third schools expecting to enroll in less advanced courses such as pre-algebra or general mathematics. At each successive level of advancement in coursework, students had higher average proficiencies, with students who anticipated enrolling in geometry having the highest average proficiency, followed by students who were planning to take algebra, who were followed by students moving into pre-algebra, who were trailed by students planning to enroll in general mathematics.

The earlier that students in middle school took their first-year algebra course, the higher their average mathematics proficiency. The positive association generally held across different demographic subgroups by race/ethnicity and gender. Part of this association is due to selection: The more able seventh and eighth graders are identified as having the mathematical skills to study algebra I rather than taking a regular mathematics class. This tracking system then permits the most able students to move into geometry in grade 9.

Table 6.1

Students' Reports on Current Mathematics Course, Grade 8

	Assessment Years	ALGEBRA Percent of Students	ALGEBRA Average Proficiency	PRE-ALGEBRA Percent of Students	PRE-ALGEBRA Average Proficiency	EIGHTH-GRADE MATHEMATICS Percent of Students	EIGHTH-GRADE MATHEMATICS Average Proficiency	OTHER MATHEMATICS Percent of Students	OTHER MATHEMATICS Average Proficiency
Grade 4									
Nation	1992	20 (1.0)	299 (1.8)	28 (2.2)>	272 (1.5)	49 (2.5)<	255 (1.3)	3 (0.4)	249 (4.1)
	1990	16 (1.1)	295 (2.5)	20 (1.8)	271 (2.3)	61 (2.0)	252 (1.4)	3 (0.4)	257 (5.3)
Top One-Third	1992	27 (2.4)	313 (1.9)	32 (3.5)	284 (1.6)	38 (3.2)	277 (1.8)>	3 (0.6)	280 (9.0)
	1990	23 (2.1)	307 (2.6)	24 (2.8)	284 (3.0)	50 (3.7)	267 (1.6)	3 (0.5)	268 (7.7)
Bottom One-Third	1992	13 (1.4)	270 (3.4)	22 (2.7)	258 (3.0)	61 (2.6)	237 (1.4)	4 (0.8)	236 (4.1)
	1990	12 (1.9)	273 (6.0)	17 (2.4)	253 (4.0)	67 (3.5)	237 (1.9)	4 (0.8)	246(11.4)
White	1992	22 (1.3)	306 (1.6)	30 (2.5)>	278 (1.2)	45 (3.1)<	265 (1.3)>	3 (0.4)	258 (5.5)
	1990	18 (1.5)	300 (2.5)	22 (2.2)	277 (2.1)	57 (2.3)	260 (1.6)	3 (0.6)	265 (6.9)
Black	1992	13 (1.7)	258 (4.8)	23 (3.7)	246 (3.1)	60 (3.8)	230 (1.4)	4 (1.2)	232 (5.4)
	1990	9 (2.1)	263 (8.9)	16 (2.9)	246 (6.0)	71 (4.5)	234 (3.2)	2 (0.8)	228(14.5)
Hispanic	1992	12 (1.2)	277 (4.3)	20 (2.5)	256 (2.6)	62 (2.8)<	240 (1.5)	5 (0.8)	231 (7.0)
	1990	7 (1.5)	276 (8.3)	13 (3.5)	260 (4.9)	74 (3.9)	240 (2.7)	4 (0.9)	230 (9.8)
Asian/Pacific Islander	1992	42 (5.1)	313 (5.1)	24 (3.4)	278 (3.5)	32 (5.1)	264 (4.6)	2 (0.8)	277(32.0)
	1990	39 (6.6)!	303 (3.6)!	22 (6.1)!	285 (5.6)!	33 (6.3)!	251 (5.4)!	6 (2.3)!	297(12.1)!
American Indian	1992	7 (2.6)	277(14.4)	30 (6.0)	258 (6.3)	57 (6.4)<	252 (3.8)	5 (1.1)	226(11.3)
	1990	6 (2.7)!	247(19.0)!	8 (6.8)!	255(19.2)	84 (5.8)!	246 (7.2)!	3 (1.9)!	193(16.9)
Male	1992	19 (1.0)	299 (2.1)	28 (2.4)>	272 (1.7)	49 (2.6)<	255 (1.4)	4 (0.3)	249 (5.6)
	1990	16 (1.2)	298 (3.0)	19 (1.7)	275 (2.8)	62 (2.1)	253 (1.6)	3 (0.4)	254 (6.1)
Female	1992	20 (1.3)	300 (2.1)	28 (2.2)	272 (1.8)	48 (2.7)<	254 (1.5)	3 (0.5)	250 (5.4)
	1990	16 (1.6)	293 (2.8)	21 (2.1)	268 (2.8)	60 (2.4)	252 (1.5)	4 (0.7)	260 (8.6)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

! Interpret with caution — the nature of the sample does not allow accurate determination of the variability of this estimated statistic.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). When the proportion of students is either 0 percent or 100 percent, the standard error is inestimable. However, percentages 99.5 percent and greater were rounded to 100 percent and percentages 0.5 percent or less were rounded to 0 percent. Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 6.2

Students' Reports on Which Mathematics Class They Plan to Take in Ninth Grade, Grade 8

Assessment Year – 1992	"I DON'T KNOW"		BASIC, GENERAL, BUSINESS OR CONSUMER MATHEMATICS		PRE-ALGEBRA		ALGEBRA I OR ELEMENTARY ALGEBRA		GEOMETRY	
	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Nation	21 (1.1)	256 (1.5)	8 (0.7)	240 (2.0)	15 (0.9)	253 (1.3)	36 (1.4)	276 (0.9)	14 (0.8)	295 (1.8)
Top One-Third	15 (2.0)	282 (3.3)	4 (0.7)	254 (4.8)	12 (1.3)	271 (2.3)	44 (2.3)	288 (0.9)	19 (1.9)	313 (1.9)
Bottom One-Third	28 (1.3)	239 (1.4)	13 (1.1)	230 (2.4)	18 (1.3)	237 (1.6)	27 (2.0)	258 (1.7)	11 (1.0)	266 (3.9)
White	20 (1.3)	265 (2.0)	7 (0.7)	250 (1.8)	14 (1.0)	262 (1.6)	40 (1.6)	282 (0.9)	15 (1.0)	305 (1.5)
Black	24 (1.5)	235 (2.1)	13 (1.6)	219 (3.3)	19 (2.0)	229 (2.5)	28 (2.9)	247 (2.3)	12 (1.2)	252 (4.5)
Hispanic	28 (1.5)	236 (2.4)	12 (1.5)	235 (3.5)	21 (2.1)	244 (2.2)	23 (1.9)	260 (2.0)	11 (1.2)	273 (4.8)
Asian/Pacific Islander	22 (3.4)	276 (6.5)	4 (1.1)	264(10.0)	7 (2.5)	242 (7.3)	33 (3.5)	282 (3.2)	31 (4.9)	313 (6.6)
American Indian	25 (2.9)	246 (4.0)	18 (3.8)	245 (6.6)	16 (3.7)	256 (6.5)	30 (4.1)	262 (6.7)	8 (2.6)	274(11.3)
Male	20 (1.0)	256 (1.9)	9 (0.8)	240 (2.5)	15 (1.0)	255 (1.7)	36 (1.6)	275 (1.1)	14 (0.8)	293 (1.9)
Female	22 (1.4)	256 (2.1)	8 (0.7)	239 (2.3)	16 (1.0)	251 (1.6)	35 (1.6)	277 (1.2)	14 (1.1)	297 (2.5)

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). When the proportion of students is either 0 percent or 100 percent, the standard error is inestimable. However, percentages 99.5 percent and greater were rounded to 100 percent and percentages 0.5 percent or less were rounded to 0 percent. The percentages may not add to 100 percent because a small number of students reported planning to take other mathematics courses.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Number of Mathematics Courses Taken in High School

Students' mathematics curricula in the U.S. have been found to be repetitive compared to those in other countries, resulting in students' having to wait until later in school to study more advanced mathematics.[49] A late start in beginning the sequence also can reduce students' chances for completing the series of advanced courses. Finally, a large proportion of students simply opt out of the mathematics pipeline during high school, and it will be difficult to substantially improve the overall national profile of mathematics achievement without increasing course enrollments.[50]

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[49] Lapointe, A. E., Mead, N. A., & Askew, J. M., *Learning Mathematics* (Princeton, NJ: The International Assessment of Educational Progress, Education Testing Service, 1992).

Robitaille, D. F., & Travers, K.J., "International Studies of Achievement in Mathematics." In D. A. Grouws, *Handbook on Mathematics Teaching and Learning* (New York, NY: MacMillan, 1992).

Stevenson, H. W. & Stigler, J. W., *The Learning Gap, Why Our Schools Are Failing and What We Can Learn from Japanese and Chinese Education* (New York, NY: Summit Books, 1992).

To supplement the eighth graders' course-taking reports, twelfth graders were asked when they took first-year algebra. As shown in Table 6.3, 23 percent of the twelfth graders reported having taken algebra before the ninth grade, 51 percent reported taking algebra in the ninth grade, and most of the rest took algebra in the tenth or eleventh grades. Six percent reported no study of algebra. Mirroring the results found at grade 8, twelfth graders in top one-third schools were twice as likely as those in bottom one-third schools to have taken algebra before ninth grade (31 versus 15 percent). In addition, the earlier students took an algebra course, the higher their average mathematics proficiency by the time they reached grade 12.

Table 6.4 presents high school seniors' reports about the amount of mathematics they had taken during grades 9 through 12. There was a change noted at the lowest end of the course-taking frequency between 1990 and 1992. The percentage of twelfth graders reporting that they had taken as little as three or fewer semesters of mathematics in high school dropped from 18 percent to 14 percent. There were similar decreases for students in the top one-third of the schools and White students.

The seniors' reports of more semesters of mathematics taken in high school were positively associated with average mathematics proficiency. This association held not only for the nation, but more importantly, across racial/ethnic categories and both genders. The encouraging information that students were taking more mathematics courses and that more course taking was reflected in higher achievement, was tempered by an inequitable distribution of course taking among racial/ethnic groups.

Fifty-eight percent of the twelfth graders in the top one-third of the schools reported taking eight semesters of mathematics compared to only 26 percent in the bottom one-third of the schools. Similarly, other substantial discrepancies in the percentages of students having taken eight semesters of high school mathematics were noted between Asian/Pacific Islander students (64 percent) and Black and Hispanic students (30 to 32 percent). The NCES *High School Transcript Study* of 1990, 1987, and 1982 high school graduates supports the NAEP findings. Students were taking more mathematics courses in 1990 than in 1982, but large differences still remained among racial/ethnic subpopulations.[51]

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[50] Steen, L., editor, *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (Washington, DC: National Research Council, National Academy Press, 1989)

Table 6.3

Students' Reports on the Grade Level at Which They Initially Took a First-Year Algebra Course, Grade 12

	BEFORE 9TH GRADE		9TH GRADE		10TH GRADE		11TH OR 12TH GRADE		HAVE NOT STUDIED ALGEBRA	
Assessment Year – 1992	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Nation	23 (1.0)	323 (1.2)	51 (1.4)	302 (0.8)	15 (0.8)	280 (1.4)	5 (0.5)	267 (1.8)	6 (0.5)	255 (2.0)
Top One-Third	31 (1.6)	335 (1.1)	54 (2.0)	314 (1.1)	11 (1.0)	291 (3.2)	2 (0.4)	278 (4.4)	2 (0.4)	267 (6.6)
Bottom One-Third	15 (1.6)	300 (2.7)	48 (2.4)	285 (1.5)	19 (1.6)	269 (2.2)	8 (1.2)	257 (2.5)	9 (1.5)	253 (3.6)
White	24 (1.1)	328 (1.1)	52 (1.5)	307 (0.7)	14 (0.8)	286 (1.6)	4 (0.5)	273 (1.8)	6 (0.5)	258 (2.1)
Black	18 (1.6)	294 (2.9)	48 (3.4)	279 (2.3)	19 (2.4)	265 (2.2)	9 (1.2)	256 (2.9)	7 (1.3)	247 (4.0)
Hispanic	17 (2.0)	300 (5.0)	45 (3.0)	291 (1.8)	23 (2.4)	273 (2.8)	9 (1.4)	261 (4.2)	7 (2.3)	250 (6.7)
Asian/Pacific Islander	40 (4.9)	335 (4.7)	44 (3.8)	307 (3.4)	10 (2.6)	288 (4.9)	4 (1.4)	286 (7.7)	2 (0.8)	284(17.7)
American Indian	23 (7.0)	312 (6.8)	36 (9.3)	288 (8.2)	20 (7.2)	260(15.3)	7 (6.6)	266(15.7)	14 (5.0)	255(16.6)
Male	24 (1.1)	324 (1.8)	49 (1.5)	304 (1.2)	15 (1.0)	283 (1.5)	5 (0.6)	272 (2.7)	7 (0.6)	258 (2.7)
Female	23 (1.1)	321 (1.6)	52 (1.6)	300 (0.9)	15 (1.0)	278 (2.0)	5 (0.6)	262 (2.3)	5 (0.5)	252 (2.4)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

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[51] Legum, S., et al., *The 1990 High School Transcript Study Tabulations: Comparative Data on Credits Earned and Demographics for 1990, 1987, and 1982 High School Graduates* (Washington, DC: National Center for Education Statistics, U.S. Department of Education, 1993).

Table 6.4

Students' Reports on the Number of Semesters of High School Mathematics Courses Taken in Grades 9 through 12, Grade 12

	Assessment Years	ZERO TO THREE SEMESTERS		FOUR TO FIVE SEMESTERS		SIX TO SEVEN SEMESTERS		EIGHT SEMESTERS OR MORE	
		Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Nation	1992	14 (0.7)<	270 (1.4)	19 (0.9)	287 (1.2)>	25 (0.9)	299 (1.0)	42 (1.2)	320 (1.0)
	1990	18 (1.1)	268 (2.0)	18 (1.0)	282 (1.7)	25 (1.4)	298 (1.4)	39 (2.0)	320 (1.4)
Top One-Third	1992	6 (0.7)<	284 (4.6)	14 (1.6)	299 (2.0)>	22 (1.4)	308 (1.7)	58 (2.1)	329 (1.3)
	1990	11 (1.3)	276 (2.9)	16 (1.6)	290 (2.2)	22 (2.9)	304 (2.4)	51 (2.8)	328 (2.3)
Bottom One-Third	1992	24 (1.6)	262 (2.2)	23 (1.6)	276 (1.8)	27 (1.5)	288 (2.0)	26 (1.3)	300 (2.5)
	1990	25 (2.4)	258 (2.6)	23 (1.9)	271 (2.5)	25 (2.0)	284 (2.9)	27 (1.6)	298 (2.6)
White	1992	12 (0.8)<	276 (1.6)	18 (0.9)	292 (1.5)>	26 (1.0)	302 (1.0)	44 (1.3)	324 (1.0)
	1990	17 (1.2)	274 (2.4)	17 (1.2)	286 (1.7)	26 (1.6)	301 (1.6)	41 (2.5)	322 (1.6)
Black	1992	21 (1.7)	255 (2.7)	27 (2.3)	272 (2.4)>	19 (1.5)	279 (2.8)	32 (2.4)	295 (2.9)
	1990	25 (2.9)	250 (3.2)	26 (2.9)	262 (3.2)	24 (2.7)	276 (3.5)	25 (3.1)	297 (4.1)
Hispanic	1992	20 (1.8)	264 (3.9)	19 (2.4)	280 (3.1)	30 (3.1)	292 (3.2)	30 (3.5)	307 (2.8)
	1990	20 (2.8)	260 (5.1)	25 (4.1)	278 (5.4)	23 (4.0)	288 (5.0)	32 (3.6)	302 (4.1)
Asian/Pacific Islander	1992	4 (1.4)	276 (6.8)	15 (3.4)	292 (4.9)	17 (2.5)	309 (3.6)	64 (4.5)	331 (4.0)
	1990	6 (3.4)	254(15.7)	11 (9.2)	311(15.5)	19 (4.1)	313 (6.9)	64 (8.3)	328 (5.1)
American Indian	1992	24 (7.5)	256(15.5)	22 (6.1)	274(19.9)	26 (9.7)	292 (8.8)	28 (8.4)	313 (6.2)
	1990	19(10.7)	236(13.0)	28(10.9)	275(16.5)	24(10.2)	304(13.3)	28(13.2)	322 (7.4)
Male	1992	13 (0.9)	272 (2.0)	20 (1.1)	290 (1.8)	24 (1.1)	301 (1.5)	43 (1.4)	323 (1.1)
	1990	17 (1.6)	270 (3.7)	18 (1.6)	284 (2.4)	20 (1.4)	299 (2.2)	45 (2.4)	322 (2.1)
Female	1992	14 (0.9)	269 (1.9)	18 (1.0)	284 (1.4)	27 (1.1)	298 (1.4)	41 (1.5)	318 (1.2)
	1990	18 (1.4)	267 (2.3)	18 (1.2)	279 (2.6)	29 (1.9)	297 (1.6)	34 (2.3)	317 (1.3)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Algebra and Calculus Coursework in High School

It is informative to study not only when students first took algebra, but also how far they advanced in their coursework sequence. Table 6.5 contains the twelfth graders' reports on the extent of their schooling in algebra. Nationally, 41 percent of the students never advanced beyond algebra I, with another 44 percent reaching algebra II, and relatively few reaching pre-calculus (11 percent) or calculus (5 percent). Students in top-third schools advanced farther than those in bottom-third schools, with 24 versus 9 percent progressing beyond algebra II. Although more Black students reported reaching calculus in 1992 than in 1990, these students still trailed the percentages of Asian/Pacific Islander students reaching this level. In 1992, 3 percent of the Black twelfth graders reported taking calculus, compared to 17 percent of the Asian/Pacific Islander students. Males and females reported similar amounts of coursework.

As might be expected, the more advanced the students' mathematics curriculum, the higher their average proficiency. This relationship tended to hold true across the nation, racial/ethnic groups, and gender.

Table 6.5

Students' Reports on Algebra and Calculus Course Taking, Grade 12

		HAVE NOT STUDIED ALGEBRA		ONLY TAKEN PRE-ALGEBRA		ONLY TAKEN ALGEBRA I	
	Assessment Years	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Nation	1992	6 (0.5)<	260 (2.4)>	6 (0.5)	269 (2.1)	29 (1.3)	286 (1.8)
	1990	9 (0.8)	251 (2.3)	8 (0.7)	264 (2.4)	28 (1.6)	285 (2.0)
Top One-Third	1992	3 (0.6)	279 (5.3)>	4 (0.7)	285 (4.5)	22 (1.9)	304 (3.6)
	1990	4 (1.1)	258 (5.3)	5 (0.7)	269 (5.7)	27 (3.4)	300 (3.3)
Bottom One-Third	1992	10 (1.4)	254 (3.9)	9 (1.0)	258 (2.3)	36 (1.7)	272 (1.7)
	1990	14 (2.1)	247 (3.5)	10 (1.5)	252 (3.3)	33 (2.1)	269 (1.6)
White	1992	5 (0.6)	262 (3.1)	5 (0.5)	273 (2.8)	27 (1.5)	293 (2.0)
	1990	8 (0.9)	256 (2.4)	7 (0.8)	270 (3.0)	27 (1.7)	291 (1.9)
Black	1992	8 (1.5)	250 (3.4)	8 (1.2)	255 (3.3)	37 (1.8)	267 (2.5)
	1990	13 (1.8)	240 (4.2)	10 (1.5)	242 (4.1)	31 (2.6)	263 (2.7)
Hispanic	1992	7 (2.1)	256 (5.8)	9 (1.2)	265 (4.3)	34 (2.6)	275 (3.0)
	1990	17 (3.3)	242 (7.0)	10 (2.2)	264 (6.3)	31 (3.2)	271 (2.9)
Asian/Pacific Islander	1992	1 (0.5)	286(17.4)	4 (1.6)	295 (7.1)	2[illegible] (3.3)	294 (4.8)
	1990	6 (1.9)	250(13.9)	10 (6.1)	275(10.1)	24 (4.8)	314 (8.7)
American Indian	1992	10 (8.2)	266(16.0)	11 (5.3)	264(10.5)	44(13.3)	276(10.6)
	1990	8 (7.1)!	260(18.7)!	5 (5.2)!	251 (5.6)	31(10.9)	271(16.6)!
Male	1992	6 (0.7)<	261 (2.9)	6 (0.6)	274 (2.4)	29 (1.3)	289 (2.0)
	1990	10 (1.2)	256 (2.5)	7 (0.7)	270 (3.4)	27 (1.7)	287 (2.1)
Female	1992	5 (0.6)	258 (3.2)>	6 (0.5)	263 (2.9)	28 (1.5)	283 (2.0)
	1990	8 (1.0)	244 (2.9)	8 (1.0)	260 (2.8)	28 (1.9)	284 (2.5)

(Table 6.5 continued on the next page)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). When the proportion of students is either 0 percent or 100 percent, the standard error is inestimable. However, percentages 99.5 percent and greater were rounded to 100 percent and percentages 0.5 percent or less were rounded to 0 percent. Percentages may not total 100 percent due to rounding error.

— Sample size insufficient to permit a reliable estimate

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 6.5

Students' Reports on Algebra and Calculus Course Taking, Grade 12 (continued)

		TAKEN ALGEBRA II BUT NOT BEYOND		TAKEN ALGEBRA III OR PRE-CALCULUS BUT NOT CALCULUS		TAKEN CALCULUS	
	Assessment Years	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency	Percent of Students	Average Proficiency
Nation	1992	44 (1.7)	308 (0.9)	11 (0.8)	324 (1.2)	5 (0.6)>	335 (2.6)
	1990	43 (1.6)	306 (1.1)	9 (1.1)	322 (2.5)	3 (0.5)	335 (5.0)
Top One-Third	1992	48 (2.8)	317 (1.1)	16 (1.2)	332 (1.6)	8 (1.3)	345 (2.9)
	1990	44 (3.6)	315 (1.9)	15 (1.7)	326 (3.5)	6 (1.1)	342 (5.4)
Bottom One-Third	1992	38 (2.1)	294 (1.7)	6 (0.8)	301 (3.9)	3 (0.6)	307 (8.1)
	1990	38 (1.7)	292 (2.4)	4 (1.0)	303 (6.1)	1 (0.5)	—
White	1992	45 (2.0)	312 (0.9)	12 (0.9)	328 (1.1)	5 (0.6)	340 (2.4)
	1990	44 (2.0)	310 (1.2)	10 (1.4)	324 (2.8)	4 (0.5)	336 (5.2)
Black	1992	38 (2.8)	288 (1.9)	7 (1.2)	299 (4.0)	3 (0.6)>	295(12.4)
	1990	39 (2.5)	284 (2.6)	6 (1.6)	300 (6.9)	0 (0.4)	—
Hispanic	1992	40 (3.9)	298 (2.7)	6 (1.0)	309 (6.3)	4 (0.8)	303(12.6)
	1990	38 (3.4)	296 (3.1)	3 (0.9)	309 (8.1)	1 (0.7)	—
Asian/Pacific Islander	1992	45 (4.7)	316 (3.5)	12 (3.2)	322 (4.3)	17 (4.0)	345 (4.2)
	1990	42 (4.3)	320 (5.6)	13 (3.9)	322 (7.1)	5 (2.9)	350 (9.4)
American Indian	1992	30 (8.5)	308(11.5)	1 (1.4)	—	4 (2.7)	327 (8.8)
	1990	40(10.4)	40(10.4)	12(12.6)!	—	4 (4.6)	—
Male	1992	42 (1.8)	311 (1.3)	11 (0.9)	326 (1.7)	5 (0.6)	322 (3.8)
	1990	41 (1.9)	309 (1.8)	10 (1.2)	328 (2.8)	4 (0.6)	334 (7.4)
Female	1992	45 (1.9)	206 (1.0)	11 (1.0)	322 (1.8)	5 (0.6)>	338 (3.0)
	1990	45 (1.7)	304 (1.0)	8 (1.2)	315 (3.2)	2 (0.5)	335 (5.2)

Geometry and Statistics Coursework in High School

The role of geometry in the American educational system has changed over the years. Some educational researchers have cited geometry as the new "gatekeeper" course for access to higher education, since most colleges are now requiring the completion of a course in geometry prior to entrance.[52]

Table 6.6 shows that the percentage of twelfth graders who had studied geometry as a separate course increased from 71 to 77 percent between 1990 and 1992. This increase in geometry course taking may be in response to the gatekeeper perception, changes in high school graduation requirements, and the possible impact of national standards. However, this increase is not equally distributed across racial/ethnic groups. Asian/Pacific Islander students were most likely to have completed geometry (87 percent), while Black students (72 percent) and Hispanic students (68 percent) were least likely to have completed a geometry course. Seventy-eight percent of the White twelfth graders had taken geometry.

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[52] Pelavin, S. & Kane, M., *Changing the Odds: Factors Increasing Access to College* (New York, NY: College Board Publications, 1990).

While the average proficiency of students who had taken geometry remained constant in 1990 and 1992, the average proficiency of students who were not enrolled in a geometry course increased during the same two-year period. This increase may be due to increased geometry content in other mathematics courses in response to new content standards.

The most striking fact from the table is the marked increase in average proficiency for those students who had taken geometry or both geometry and trigonometry relative to those who had not taken a geometry course. The average proficiency of students without a geometry course was 270, compared with average proficiencies of 306 for those with a geometry course and 318 for those with both geometry and trigonometry. The large increase in average proficiency for those who had taken a geometry course held across all racial/ethnic groups and both genders. However, the increase was significantly larger for students in top one-third schools than for those in bottom one-third schools.

Table 6.7 shows that although statistics was not a popular course, this was slowly changing, with 12 percent of twelfth graders having taken statistics in 1990, rising to 17 percent in 1992.

Table 6.6

Students' Reports on Geometry and Trigonometry Course Taking, Grade 12

		HAVE NOT STUDIED GEOMETRY		TAKEN GEOMETRY		TAKEN GEOMETRY AND TRIGONOMETRY	
	Assessment Years	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Nation	1992	23 (1.3)<	270 (1.4)>	57 (1.5)	306 (1.0)	20 (1.2)	318 (1.1)
	1990	29 (1.6)	264 (1.2)	55 (1.7)	304 (1.1)	16 (1.5)	318 (2.0)
Top One-Third	1992	13 (1.6)	285 (3.0)>	62 (2.8)	320 (1.4)>	25 (2.4)	326 (1.9)
	1990	17 (2.0)	275 (2.0)	58 (3.3)	314 (1.5)	25 (2.8)	326 (2.1)
Bottom One-Third	1992	36 (3.4)	262 (2.2)	51 (3.0)	288 (1.7)	13 (1.5)	301 (2.9)
	1990	43 (3.1)	255 (2.6)	46 (2.3)	286 (2.4)	11 (1.8)	300 (4.1)
White	1992	22 (1.3)	274 (1.5)	58 (1.6)	311 (1.0)	20 (1.5)	322 (1.2)
	1990	26 (1.8)	270 (1.3)	57 (2.0)	309 (1.2)	17 (1.7)	323 (2.0)
Black	1992	28 (3.5)	255 (2.4)>	54 (3.6)	280 (1.9)	18 (2.1)	296 (3.1)
	1990	39 (3.1)	245 (2.7)	47 (3.3)	280 (2.6)	14 (1.8)	293 (4.8)
Hispanic	1992	32 (5.4)	264 (2.5)	55 (6.9)	295 (1.5)	13 (2.2)	301 (5.9)
	1990	42 (3.9)	255 (5.0)	50 (3.4)	291 (2.4)	8 (1.8)	301 (7.8)
Asian/Pacific Islander	1992	13 (2.6)	286 (4.3)	53 (4.6)	320 (3.9)	34 (4.4)	322 (4.2)
	1990	15 (4.7)	267 (8.2)	63 (4.9)	321 (3.9)	22 (8.6)	323 (7.2)
American Indian	1992	51 (7.4)	265(11.3)	44 (7.9)	302 (7.2)	5 (3.2)	330 (5.3)
	1990	40(15.0)	269(14.0)	33(12.2)	302 (7.0)	26(13.2)	298(27.2)
Male	1992	24 (1.6)	273 (1.8)>	54 (1.7)	308 (1.2)	21 (1.3)	321 (1.7)
	1990	29 (1.8)	267 (1.3)	51 (2.2)	307 (1.4)	20 (1.8)	319 (2.3)
Female	1992	22 (1.3)<	267 (1.5)>	59 (1.7)	304 (1.1)	19 (1.4)>	315 (1.5)
	1990	29 (1.9)	260 (2.0)	58 (1.9)	302 (1.3)	13 (1.4)	317 (2.4)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details).

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 6.7

Students' Reports on Statistics Course Taking, Grade 12

		HAVE NOT STUDIED STATISTICS		TAKEN STATISTICS	
	Assessment Years	Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Nation	1992	83 (0.7)<	299 (0.9)>	17 (0.7)>	307 (2.1)
	1990	88 (1.2)	292 (1.1)	12 (1.1)	306 (3.2)
Top One-Third	1992	80 (1.7)	315 (1.1)>	20 (1.5)	324 (2.6)
	1990	84 (2.2)	308 (1.1)	16 (2.0)	320 (4.2)
Bottom One-Third	1992	84 (1.1)	280 (1.1)>	16 (1.2)	282 (3.7)
	1990	89 (2.2)	273 (1.8)	11 (1.9)	282 (5.9)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details).

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

High School Seniors Not Taking Mathematics

Unfortunately, for many high school seniors, questions about the level of their current mathematics curriculum are moot. As Table 6.8 shows, fewer than two-thirds of all high school seniors (63 percent) reported that they were enrolled in any type of mathematics course. Given that only 57 percent of the students reported having taken a geometry course, and only 16 percent claimed to have advanced to pre-calculus or higher, many of the 37 percent of students who had left mathematics by grade 12 were never exposed to advanced mathematics in high school. As might be anticipated, those students not taking a mathematics course at the time of the assessment did not perform as well as the students enrolled in a mathematics class.

Table 6.8

Students' Reports on Currently Taking a Mathematics Course, Grade 12

	Assessment Years	YES		NO	
		Percentage of Students	Average Proficiency	Percentage of Students	Average Proficiency
Are you taking a mathematics class this year?					
Nation	1990	63 (1.2)	306 (1.0)	37 (1.2)	286 (1.1)>
	1992	59 (2.0)	304 (1.3)	41 (2.0)	281 (1.0)
Top One-Third	1992	73 (1.9)	322 (1.5)	27 (1.9)	301 (1.6)>
	1990	67 (2.7)	319 (1.6)	33 (2.7)	292 (1.7)
Bottom One-Third	1992	56 (1.9)	284 (1.5)	44 (1.9)	273 (1.3)
	1990	53 (2.1)	280 (1.7)	47 (2.1)	268 (1.6)

> The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level.

< The value for 1992 was significantly lower than the value for 1990 at about the 95 percent confidence level.

The standard errors of the estimated percentages and proficiencies are presented in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). When the proportion of students is either 0 percent or 100 percent, the standard error is inestimable. However, percentages 99.5 percent and grater were rounded to 100 percent and percentages 0.5 percent or less were rounded to 0 percent.

Summary

Between 1990 and 1992 there was a significant increase in the percentage of eighth graders taking pre-algebra and a significant decrease in the percentages taking a general eighth-grade mathematics course. In 1992, roughly half (49 percent) of all eighth graders were enrolled in general mathematics courses. The students in pre-algebra (28 percent) and algebra (20 percent) courses had higher average proficiencies than general mathematics students, as did eighth graders planning to progress to the study of geometry or algebra in ninth grade. Both eighth and twelfth graders' reports on course taking indicated that the earlier students were exposed to more advanced coursework, the higher their average proficiencies. Often, this is a result of selection, with the more able students tracked into a sequence of pre-algebra in grade 7, algebra in grade 8, and geometry in grade 9.

Twelfth graders reported somewhat more mathematics course taking in 1992 than in 1990. The percentage reporting only zero to three semesters of mathematics coursework during their high school years decreased from 18 to 14 percent. Large proportions of high school students, however, do not take four years of mathematics in grades 9 through 12. Forty-two percent of the high school seniors reported taking eight semesters of mathematics coursework. There were large discrepancies in the amount of mathematics coursework reported among various subpopulations. For example, 58 percent of the students in the top one-third schools reported eight semesters of mathematics coursework compared to 26 percent in the bottom one-third schools. There was a direct positive relationship between more coursework and mathematics achievement.

The farther students went in studying algebra, geometry, and statistics, the higher their performance on the assessment. In recent years, geometry has become the "gatekeeper" course for access to higher education, a fact that is borne out by the wide performance disparity between the 77 percent of students who had taken geometry and the 23 percent who had not. Despite the high goals set for mathematics achievement in the United States — becoming number one in the world by the year 2000 — fewer than two-thirds of the high school seniors were enrolled in a mathematics course of any type.

Procedural Appendix

Overview of Procedures Used in NAEP's 1992 Mathematics Assessment

The 1992 Assessment Framework and Questions

The framework underlying both the 1990 and 1992 mathematics assessments was initially developed for the 1990 assessment and subsequently approved for use in both assessments by the National Assessment Governing Board. It was developed through a consensus process managed by the Council of Chief State School Officers, and the items were developed through a similarly broad-based process managed by Educational Testing Service. The development of the mathematics assessments benefited from the involvement of hundreds of representatives from State Education Agencies who attended numerous NETWORK meetings; served on committees; reviewed the framework, objectives, and questions; and in general, provided important suggestions on all aspects of the program.

The mathematics assessment framework is a five-by-three matrix specifying five content areas — Numbers and Operations; Measurement; Geometry; Data Analysis, Statistics, and Probability; and Algebra and Functions, plus three process or ability areas. These include Conceptual Understanding, Procedural Knowledge, and Problem Solving.[53] Consistent with standards developed by the National Council of Teachers of Mathematics, many questions required students to construct their responses and some questions asked for explanations of their reasoning.[54]

The Assessment Design

For the measure of mathematics achievement used in this report, each student received a booklet containing a set of general background questions, a set of subject-specific background questions, three 15-minute segments, or blocks, of cognitive items, and a set of questions about his or her motivation and familiarity with the assessment material. Students received different blocks of cognitive items in their booklets according to a careful plan. The 1992 assessment was based on an adaptation of matrix sampling called balanced incomplete block (BIB) spiraling — a design that enables broad coverage of mathematics content while minimizing the burden for any one student. The balanced incomplete block part of the design assigns blocks of items to booklets and each pair of blocks appears together in at least one booklet. The spiraling part of the method cycles the booklets for administration, so that typically only a few students in any assessment session receive the same booklet.

In accordance with this design, there were 13 blocks presented in 26 booklets. Each block appeared in exactly six booklets, and each block appeared with every other block in at least one booklet. Students at grades 4 and 8 were given calculators to use with three of the 13 blocks and were trained in their use prior to the assessment. Students at grade 12 were given calculators to use with four of the 13 blocks. At the fourth grade, students were provided with four-function calculators and at grades 8 and 12, they were provided with scientific calculators. For another block, fourth-grade students were provided with a ruler, and eighth- and twelfth-grade students with a protractor/ruler. For still another block, at all three grades, students were given geometric shapes (manipulatives) to provide a concrete basis for determining their answers.

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[53] *Mathematics Objectives, 1990 Assessment* (Princeton, NJ: National Assessment of Educational Progress, Educational Testing Service, 1988).

[54] Dossey, J. A., Mullis, I. V. S., & Jones, C. O., *Can Students Do Mathematical Problem Solving? Results from Constructed Response Questions in NAEP's 1992 Mathematics Assessment* (Washington, DC: National Center for Education Statistics, Government Printing Office, 1993).

Background Questionnaires

As part of the 1990 and 1992 mathematics assessments, students, teachers, and school administrators completed background questionnaires. Questionnaires were given to the mathematics teachers of the fourth- and eighth-grade students participating in the assessment and to the principal or another administrator in each participating school. An expert panel knowledgeable about NAEP, educational policy, and instruction in the curriculum areas being assessed in 1992 developed guidelines for the student, school, and teacher questionnaires. The framework focused on five educational areas: instructional content, instructional practices and experiences, teacher characteristics, school conditions and contexts, and conditions beyond school (i.e., home support, out-of-school activities, and attitudes).[55] The outline for the background questionnaire framework follows.

NAEP 1992 BACKGROUND QUESTIONNAIRE FRAMEWORK

1.0 INSTRUCTIONAL CONTENT

  1.1 course offerings in selected subject areas

  1.2 course taking in selected subject areas

  1.3 objectives, topics, and skills covered

  1.4 emphasis on facts, concepts, and higher-order skills

2.0 INSTRUCTIONAL PRACTICES AND EXPERIENCES

  2.1 assignment to classes according to ability or achievement

  2.2 grouping within classes according to ability or achievement

  2.3 teacher's freedom within the classroom

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[55] *National Assessment of Educational Progress, 1992 Policy Information Framework* (Princeton, NJ: National Assessment of Educational Progress, Educational Testing Service, 1992).

2.4 time spent on content-related instruction

2.5 use of whole class, group, and individual instruction

2.6 mode of instruction — lecture, demonstration, discussion, etc.

2.7 availability and use of materials — textbooks, supplementary materials, workbooks, kits

2.8 availability and use of equipment — computers and calculators

2.9 classroom activities and assignments

2.10 student assessment

2.11 amount of homework assigned

3.0 TEACHER CHARACTERISTICS

3.1 type of certification

3.2 highest academic degree

3.3 undergraduate and graduate course work in mathematics, reading or writing and in the teaching of those subjects

3.4 undergraduate and graduate major and minor field

3.5 in-service training in mathematics, reading, or writing and in the teaching of those subjects

3.6 other teacher development activities

3.7 number of years teaching experience in general

3.8 number of years teaching in a field

3.9 comfort in teaching mathematics

4.0 SCHOOL CONDITIONS AND CONTEXT

4.1 instructional time and teacher-pupil ratio

4.2 school-wide programs

4.3 characteristics and experience of the principal

4.4 characteristics and experience of the teaching staff

4.5 school climate

4.6 resources for students with special needs

4.7 resources for teachers

4.8 community and parental involvement

5.0 CONDITIONS BEYOND SCHOOL

5.1 language in home

5.2 country of birth

5.3 student mobility

5.4 home resources

5.5 parental support

5.6 experiences before starting school

5.7 out-of-school activities — reading

5.8 computer use

5.9 disposition to learning — attitudes toward subjects, self-confidence in subjects, value and utility of subjects, educational and vocational aspirations

Because the sampling for the teacher questionnaires was based on participating students, the responses to the mathematics teacher questionnaire do not necessarily represent all fourth- or eighth-grade mathematics teachers in the nation, or in a state or territory. Rather, they represent teachers of the representative sample of students assessed. It is important to note that in this report, as in all NAEP reports, the student is always the unit of analysis, even when information from the teacher or school questionnaire is being reported. Using the student as the unit of analysis makes it possible to describe the instruction received by representative samples of students. Although this approach may provide a different perspective from that obtained by simply collecting information from teachers or schools, it is consistent with NAEP's goal of providing information about the educational context and performance of students.

National Sampling

Sampling and data collection activities for both the 1990 and 1992 NAEP assessments were conducted by Westat, Inc. In 1992, the assessment was conducted from January through March, with some make-up sessions in early April. In 1990, the sample at each grade consisted of two equivalent half samples. The assessment was administered to the first half sample in the January to mid-March time frame, while it was administered to the second half sample in the mid-March to mid-May time frame. The first half sample from 1990 was used for trend purposes to provide a more precise basis for comparison in terms of the time of year of the assessment.

As with all NAEP national assessments, the results for the national samples were based on a stratified, three-stage sampling plan. The first stage included defining geographic primary sampling units (PSUs), which are typically groups of contiguous counties, but sometimes a single county; classifying the PSUs into strata defined by region and community type; and randomly selecting PSUs. For each grade, the second stage included listing, classifying, and randomly selecting schools, both public and private, within each PSU selected at the first stage. The third stage involved randomly selecting students within a school for participation. Some students who were selected (about 7 to 8 percent) were excluded because of limited English proficiency or severe disability.

Table A.1 presents the student and school sample sizes and the cooperation and response rates for the national assessment.

Table A.1

1992 Student and School Sample Sizes

	NUMBER OF PARTICIPATING SCHOOLS		PERCENT OF SCHOOLS PARTICIPATING		NUMBER OF STUDENTS		PERCENT OF STUDENT COMPLETION	
	1990	1992	1990	1992	1990	1992	1990	1992
Grade								
4	527	527	88	86	8,902	8,738	93	93
8	406	587	87	84	8,888	9,432	89	89
12	304	468	81	81	8,862	8,499	81	81
Total	1,237	1,582			26,652	26,669		

Although sampled schools that refused to participate were occasionally replaced, school cooperation rates were computed based on the schools originally selected for participation in the assessments. The rates, which are based on schools sampled for all subjects assessed in 1992 (reading, writing, and mathematics) and 1990 (reading, science, and mathematics), are also the best estimates for the mathematics assessment. The student completion rates represent the percentage of students assessed of those invited to be assessed in mathematics, including those assessed in follow-up sessions, when necessary.

Note: In 1992, NAEP also conducted a voluntary Trial State Assessment Program in mathematics at grades 4 and 8. The 1990 Trial State Program was conducted at grade 8. Data for the 44 jurisdictions participating in the 1992 program can be found in the *NAEP 1992 Report Card for the Nation and the States,* which provides overall achievement results for various demographic subgroups and *Can Students Do Mathematical Problem Solving?,* which looks at the results for questions where students were asked to construct their responses. A relatively complete set of achievement and background data can be found in the *Data Compendium for the NAEP 1992 Mathematics Assessment of the Nation and the States.*

Excluded Students

It is NAEP's intent to assess all selected students. Therefore, all selected students who are capable of participating in the assessment should be assessed. However, some students sampled for participation in NAEP are excluded from the sample according to carefully defined criteria. Specifically, some of the students identified as having Limited English Proficiency (LEP) or having an Individualized Education Plan (IEP) may be incapable of participating meaningfully in the assessment. These students are identified as follows:

LEP students may be excluded if:

- The student is a native speaker of a language other than English, AND
- He or she has been enrolled in an English-speaking school for less than two years, AND
- The student is judged to be incapable of taking part in the assessment.

IEP students may be excluded if:

- The student is mainstreamed less than 50 percent of the time in academic subjects and is judged to be incapable of taking part in the assessment, OR
- The IEP team has determined that the student is incapable of taking part meaningfully in the assessment.

When there is doubt, the student is included in the assessment.

For each student excluded from the assessment, school personnel completed a questionnaire about the characteristics of that student and the reason for exclusion. Approximately 7 to 8 percent of the students nationally were excluded from the assessments.

Data Collection and Scoring

As with all NAEP assessments, data collection for the 1990 and 1992 assessments was conducted by a trained field staff. For the national assessment, this was accomplished by Westat staff.

Materials collected as part of the 1992 assessment were shipped to National Computer Systems in Iowa City for processing. Receipt and quality control were managed through a sophisticated bar-coding and tracking system. After all appropriate materials were received from a school, they were forwarded to the professional scoring area, where the responses to the open-ended items were evaluated by trained staff using guidelines prepared by NAEP. Each open-ended question had a unique scoring guide that defined the criteria to be used in evaluating students' responses. Of the regular constructed-response items, most were scored right/wrong, but some included several different categories of correct and incorrect responses. The extended constructed-response questions were evaluated on a scale of 1 to 5, permitting degrees of partial credit to be given.

For the national mathematics assessment and the Trial State Assessment Program approximately 4 million student responses were scored, including a 20 percent reliability sample. The overall percentage of agreement between readers for both the national and trial state assessment reliability samples at each of the three grades assessed was 94 percent. For the constructed-response questions contained in the trend blocks, training was conducted using materials and scoring guides identical to those used for the 1990 assessment. To provide information about reliability between assessment years, 100 booklets from each of the 40 jurisdictions that participated in the 1990 Trial State Assessment Program were chosen at random to be scored again in 1992. Based on the 4,000 responses to each of the 25 questions rescored in 1992, the exact percentage of agreement was 96 percent. Subsequent to the professional scoring, the booklets were scanned, and all information was transcribed to the NAEP database at ETS. Each processing activity was conducted with rigorous quality control.

Data Analysis and IRT Scaling

After the assessment information had been compiled in the database, the data were weighted according to the population structure. The weighting for the national and state samples reflected the probability of selection for each student as a result of the sampling design, adjusted for nonresponse. Through poststratification, the weighting ensured that the representation of certain subpopulations corresponded to figures from the U.S. Census and the Current Population Survey.[56]

Analyses were then conducted to determine the percentages of students who gave various responses to each cognitive and background question. Item response theory (IRT) was used to estimate average scale-score proficiency for the nation, various subgroups of interest within the nation, and for the states and territories. IRT models the probability of answering an item correctly as a mathematical function of proficiency or skill. The main purpose of IRT analysis is to provide a common scale on which performance can be compared across groups, such as those defined by grades, and subgroups, such as those defined by race/ethnicity or gender. Because of the BIB-spiraling design used by NAEP, students do not receive enough questions about a specific topic to provide reliable information about individual performance. Traditional test scores for individual students, even those based on IRT, would lead to misleading estimates of population characteristics, such as subgroup means and percentages of students at or above a certain proficiency level. Instead, NAEP constructs sets of plausible values designed to represent the distribution of proficiency in the population. A plausible value for an individual is not a scale score for that individual but may be regarded as a representative value from the distribution of potential scale scores for all students in the population with similar characteristics and identical patterns of item response. Statistics describing performance on the NAEP proficiency scale are based on these plausible values. They estimate values that would have been obtained had individual proficiencies been observed — that is, had each student responded to a sufficient number of cognitive items so that proficiency could be precisely estimated.[57]

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[56] For additional information about the use of weighting procedures in NAEP, see Eugene G. Johnson, "Considerations and Techniques for the Analysis of NAEP Data" in *Journal of Educational Statistics* (December 1989).

For the 1992 assessment, a scale ranging from 0 to 500 was created to report performance for each content area (Numbers and Operations; Measurement; Geometry; Data Analysis, Statistics, and Probability; Algebra and Functions) and for the estimation skill area. The scales summarize examinee performance across all three question types used in the assessment (multiple-choice, regular constructed-response, and extended-response). In producing the scales, three distinct IRT models were used. Multiple-choice items were scaled using the three-parameter logistic (3PL) model; regular constructed-response questions were scaled using the two-parameter logistic (2PL) model; and the extended-response tasks were scaled using a generalized partial-credit (GPC) model (Muraki, 1992).[58] For each of the 1992 content area scales, separate scales were fit within each grade and then linked to the previously established 1990 scales via a common population linking procedure.[59] Each scale was based on the distribution of student performance across all three grades assessed in the 1990 national assessment (grades 4, 8, and 12) and had a mean of 250 and a standard deviation of 50. A composite scale was created as an overall measure of students' mathematics proficiency. The composite scale was a weighted average of the five content-area scales, where the weight for each content area was proportional to the relative importance assigned to the content area in the specifications developed by the Mathematics Objectives Panel.

As described earlier, the NAEP proficiency scales make it possible to examine relationships between students' performance and a variety of background factors measured by NAEP. The fact that a relationship exists between achievement and another variable, however, does not reveal the underlying cause of the relationship, which may be influenced by a number of other variables. Similarly, the assessments do not capture the influence of unmeasured variables. The results are most useful when they are considered in combination with other knowledge about the student population and the educational system, such as trends in instruction, changes in the school-age population, and societal demands and expectations.

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[57] For theoretical justification of the procedures employed, see Robert J. Mislevy, "Randomization-Based Inferences About Latent Variables from Complex Samples," *Psychometrika*, 56(2), 177-196, 1988).

For computational details, see *Focusing the New Design: NAEP 1988 Technical Report* (Princeton, NJ: Educational Testing Service, National Assessment of Educational Progress, 1990) and the *1990 NAEP Technical Report*.

[58] Muraki, E., "A Generalized Partial Credit Model: Application of an EM Algorithm," *Applied Psychological Measurement*, 16(2), 159-176, 1992.

[59] Yamamoto, K. & Mazzeo, J., "Item Response Theory Scale Linking in NAEP," *Journal of Educational Statistics*, Vol. 17, 155-73, 1992.

Estimating Variability

Because the statistics presented in this report are estimates of group and subgroup performance based on samples of students, rather than the values that could be calculated if every student in the nation answered every question, it is important to have measures of the degree of uncertainty of the estimates. Two components of uncertainty are accounted for in the variability of statistics based on proficiency: the uncertainty due to sampling only a relatively small number of students and the uncertainty due to sampling only a relatively small number of mathematics questions. The variability of estimates of percentages of students having certain background characteristics or answering a certain cognitive question correctly is accounted for by the first component alone.

In addition to providing estimates of percentages of students and their proficiency, this report also provides information about the uncertainty of each statistic. Because NAEP uses complex sampling procedures, conventional formulas for estimating sampling variability that assume simple random sampling are inappropriate and NAEP uses a jackknife replication procedure to estimate standard errors. The jackknife standard error provides a reasonable measure of uncertainty for any information about students that can be observed without error, but each student typically responds to so few items within any content area that the proficiency measurement for any single student would be imprecise. In this case, using plausible values technology makes it possible to describe the performance of groups and subgroups of students, but the underlying imprecision that makes this step necessary adds an additional component of variability to statistics based on NAEP proficiencies.[60]

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[60] For further details, see Eugene G. Johnson, "Considerations and Techniques for the Analysis of NAEP Data" in *Journal of Educational Statistics* (December 1989).

The reader is reminded that the standard error estimates provided with the statistics in this report appropriately take into account uncertainty due to sampling, and due to imprecision of individual measurement. NAEP results, like those from all surveys, are also subject to other kinds of errors including the effects of necessarily imperfect adjustment for student and school nonresponse and other largely unknowable effects associated with the particular instrumentation and data collection methods used. Nonsampling errors can be attributed to a number of sources: inability to obtain complete information about all selected students in all selected schools in the sample (some students or schools refused to participate, or students participated but answered only certain items); ambiguous definitions; differences in interpreting questions; inability or unwillingness to give correct information; mistakes in recording, coding, or scoring data; and other errors of collecting, processing, sampling, and estimating missing data. The extent of nonsampling errors is difficult to estimate. By their nature, the impacts of such error cannot be reflected in the data-based estimates of uncertainty provided in NAEP reports.

Drawing Inferences from the Results

The use of *confidence intervals*, based on the standard errors, provides a way to make inferences about the population means and proportions in a manner that reflects the uncertainty associated with the sample estimates. An estimated sample mean proficiency ± 2 standard errors represents a 95 percent confidence interval for the corresponding population quantity. This means that with approximately 95 percent certainty, the average performance of the entire population of interest is within ± 2 standard errors of the sample mean.

As an example, suppose that the average mathematics proficiency of students in a particular group was 256, with a standard error of 1.2. A 95 percent confidence interval for the population quantity would be as follows:

$$\text{Mean} \pm 2 \text{ standard errors} = 256 \pm 2 \bullet (1.2) = 256 \pm 2.4 = 256 - 2.4 \text{ and } 256 + 2.4 = 253.6, 258.4$$

Thus, one can conclude with 95 percent certainty that the average proficiency for the entire population of students in that group is between 253.6 and 258.4.

Similar confidence intervals can be constructed for percentages, provided that the percentages are not extremely large (greater than 90) or extremely small (less than 10). For extreme percentages, confidence intervals constructed in the above manner may not be appropriate. However, procedures for obtaining accurate confidence intervals are quite complicated. Thus, comparisons involving extreme percentages should be interpreted with this in mind.

To determine whether there is a real difference between the mean proficiency (or proportion of a certain attribute) for two groups in the population, one needs to obtain an estimate of the degree of uncertainty associated with the difference between the proficiency means or proportions of these groups for the sample. This estimate of the degree of uncertainty — called the standard error of the difference between the groups — is obtained by taking the square of each group's standard error, summing these squared standard errors, and then taking the square root of this sum.

Similar to the manner in which the standard error for an individual group mean or proportion is used, the standard error of the difference can be used to help determine whether differences exist between groups in the population. The difference between the mean proficiency or proportion of the two groups ± 2 standard errors of the difference represents an approximate 95 percent confidence interval. If the resulting interval includes zero, there is insufficient evidence to claim a real difference between groups in the population. If the interval does not contain zero, the difference between groups is statistically significant (different) at the .05 level.

The procedures described in this section, and the certainty ascribed to intervals (e.g., a 95 percent confidence interval) are based on statistical theory that assumes that only one confidence interval or test of statistical significance is being performed. When one considers sets of confidence intervals, like those for the average proficiency of all participating states and territories, statistical theory indicates that the certainty associated with the entire set of intervals is less than that attributable to each individual comparison from the set. If one wants to hold the certainty level for a specific set of comparisons at a particular level (e.g., .95), adjustments (called multiple-comparisons procedures) need to be made.

The standard errors for means and proportions reported by NAEP are statistics and subject to a certain degree of uncertainty. In certain cases, typically when the standard error is based on a small number of students or when the group of students is enrolled in a small number of schools, the amount of uncertainty associated with the standard errors may be quite large. Throughout this report, estimates of standard errors subject to a large degree of uncertainty are designated by the symbol "!". In such cases, the standard errors — and any confidence intervals or significance tests involving these standard errors — should be interpreted cautiously.

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